



Life-Cycle Consumption and Children

Jørgensen, Thomas Høgholm

Publication date:
2014

Document version
Early version, also known as pre-print

Citation for published version (APA):
Jørgensen, T. H. (2014). *Life-Cycle Consumption and Children*. Centre for Applied Microeconometrics. Department of Economics, University of Copenhagen. CAM working papers Vol. 2014 No. 02
<http://www.econ.ku.dk/cam/wp0910/>



CAM

Centre for Applied
Microeconometrics

Department of Economics
University of Copenhagen

<http://www.econ.ku.dk/CAM/>

Working paper no. 2014-02

Life-Cycle Consumption and Children

Thomas H. Jørgensen

Life-Cycle Consumption and Children

Thomas H. Jørgensen[†]

January 21, 2014

Abstract

I show that conventional estimators based on the consumption Euler equation, extensively used in studies of intertemporal consumption behavior, produce inconsistent estimates of the effect of children on consumption if potentially binding credit constraints are ignored. As a more constructive contribution, I supply a tractable approach to obtaining bounds on the effect of children and a structural estimation strategy when households face constraints. Finally, I estimate the effect of children on consumption using the Panel Study of Income Dynamics (PSID) for the US and high quality Danish administrative register data. Results suggest that children does not affect household consumption in the same magnitude previously assumed.

Keywords: Consumption, Children, Life Cycle, Credit Constraints, Structural Estimation.

JEL-Codes: D12, D14, D91, E21.

[†]Department of Economics, University of Copenhagen, Øster Farimagsgade 5, building 26, DK-1353 Copenhagen, and Centre for Applied Microeconometrics (CAM). Webpage: www.tjeconomics.com. Contact: thomas.h.jorgensen@econ.ku.dk.

I am grateful to Mette Ejrnæs and Jeppe Druedahl for fruitful discussions. I also wish to thank Bertel Schjerning, Martin Browning, Christopher Carroll, John Rust, Miles Kimball, Kjetil Storesletten, and participants at the 28th Congress of the European Econometric Association (EEA) in Gothenburg 2013, and the Center for Applied Microeconometrics (CAM) December workshop in Copenhagen 2013 for valuable comments and suggestions. The usual disclaimer applies.

1 Introduction

This paper is concerned with the effect of children on non-durable consumption over the life cycle. The empirical age profile of household consumption forms an inverse u-shape (hump) and the average number of children shares this profile leading to the intuitive reasoning that children are likely to increase household consumption. The same consumption profile can, however, be rationalized by income growth and credit constraints with very different policy implications. If imperfect markets such as constraints on credit are the main driver of the observed consumption behavior, reducing these imperfections would increase welfare. On the contrary, if the consumption behavior is due to children, resources are simply redistributed within households towards periods with a higher marginal value of consumption and no political intervention is needed. The extent to which children affect consumption behavior has, therefore, received great attention the last two decades with large effects of children on consumption being the most common finding.¹

The present study offers four significant contributions to this literature. First, I show how conventional methods, used intensively in the literature on intertemporal consumption behavior, produce inconsistent estimates of the effect of children on consumption if consumers face possibly binding credit constraints. This inconsistency of the effect of children has, to the best of my knowledge, not been documented before and, if ignored, pose a serious problem when analyzing intertemporal consumption behavior.² Secondly, I supply a tractable approach to obtaining bounds on the effect of children. Thirdly, I propose the use of a structural estimation strategy and, Finally, I estimate the effect of children on consumption using the Panel Study of Income Dynamics (PSID) for the US and high quality Danish administrative register data.

There is both a positive and negative directed bias of Euler equation estimators of the effect of children on consumption. Specifically, if the effect of children on consumption is *large*, the credit constraint likely restrain households from increasing consumption as much as what would have been desired had (additional) borrowing been possible. Imagine an extreme case where a household has so few resources that consumption tracks income perfectly in the early part of life since borrowing is not allowed. At the arrival of a child, the credit constraint forces household expenditure to be unaffected even if such a household would want to increase consumption in the presence of children, producing a downward bias. To the contrary, if the effect of children is relatively low, conventional methods will over estimate the effect of children. The reason is that, even if children does not affect consumption, early life income growth together with the credit constraint produce a positive correlation between consumption growth and changes in household demographics because children often arrive while households are young and affected by credit constraints the most.

I suggest a tractable approach to uncover bounds on the effect of children when credit

¹[Thurow \(1969\)](#) might be one of the first studies investigating the age profile of consumption to mention both children and constraints as potential explanations for the hump. Some important contributions to the literature on the effect of children are due to [Browning, Deaton and Irish \(1985\)](#); [Blundell, Browning and Meghir \(1994\)](#); [Attanasio and Weber \(1995\)](#); [Attanasio and Browning \(1995\)](#); [Attanasio, Banks, Meghir and Weber \(1999\)](#); [Fernández-Villaverde and Krueger \(2007\)](#) and [Browning and Ejrnæs \(2009\)](#).

²The fact that ignoring credit constraints produce biased Euler equation estimates is not new. [Adda and Cooper \(2003\)](#) show how Euler equation estimation of the intertemporal elasticity of substitution is upwards biased if credit constraints are ignored. However, how ignored constraints affect the estimated effect of children on consumption has *not* been thoroughly analyzed.

constraints might affect consumers. Utilizing the opposite directed biases and using simple reduced form methods with suitable instruments, I argue that young households together with suitable instruments can be used to uncover an upper bound while older households can be used to estimate a lower bound.

Additionally, I propose a flexible structural estimation strategy in which the economic environment of households is fully specified and the resulting model is solved numerically. This method relies on more structure but overcomes limitations of the bounds estimation approach allowing for measurement error in key variables, simultaneous estimation of *all* relevant structural parameters, and arbitrary number, age and scale effect of children on consumption. The general estimation framework does not rely on availability of panel data and the approach is, therefore, extremely useful since repeated cross section data, which most surveys are, can be used to estimate parameters of interest in this framework.

Finally, I estimate the effect of children on consumption using both the Panel Study of Income Dynamics (PSID) and high quality Danish administrative register data. Results suggest, for both data sources, that the effect of children is lower than previously assumed in the literature.³ The estimated bounds suggest that what is typically reported in the literature is close to the upper bound, while I estimate effects close to the lower bound, using the proposed structural estimation approach. The PSID contains only food consumption measures but [Blundell, Pistaferri and Preston \(2008\)](#) imputes total non-durable consumption for PSID households based on food consumption in the Consumer Expenditure Survey (CEX) which I use. The Danish register data contain information on household income and wealth along with household characteristics providing high quality longitudinal information, compared to the more noisy recall survey data in the PSID.⁴

In stark contrast to what I find, it seems broadly accepted that children play an important role in generating the observed consumption profiles. In an influential study by [Attanasio, Banks, Meghir and Weber \(1999\)](#), the number of children is found to be important in order to describe the consumption behavior of US consumers, using the CEX. This is supported by the results in [Attanasio and Browning \(1995\)](#) using the UK Family Expenditure Survey (FES). [Fernández-Villaverde and Krueger \(2007\)](#) argue that around 50 percent of the hump in the consumption profile in the CEX is due to household demographics while [Browning and Ejrnæs \(2009\)](#) argue that the number and age of children can explain completely the hump in consumption. All existing studies apply Euler equation estimation techniques ignoring potentially binding credit constraints. If the effect of children is relatively low ignoring credit constraints produce a positive bias in Euler equation estimators offering a potential explanation for why existing empirical studies find large effects of children on consumption.

Credit constraints cannot be ruled out to affect consumers. Recent research suggest that observed behavior of especially young consumers is likely due to households being credit constrained ([Leth-Petersen, 2010](#)). The empirical importance of constraints are, however, still an

³For some specifications, I estimate an effect of children on consumption close to the estimate in [Attanasio, Banks, Meghir and Weber \(1999\)](#) for US consumers. The estimates are, however, not statistically significant and produce average consumption profiles that do not fit the data as well as a model in which children does not affect consumption.

⁴[Runkle \(1991\)](#) finds that more than 70 percent of the variation in changes in log food consumption in the PSID is measurement error.

open question since testing for the presence of credit constraints is an extremely challenging task due to the unobservable shadow prices of additional resources. In fact, if there is no constraints on credit but instead a probability of a very low income (zero income in unemployment, say) many of the results in the present study still holds. This is because the combination of income growth, risk averse consumers, and a probability of a very low income induce a self-imposed credit constraint and consumer behavior very similar to that of consumers facing an “explicit” credit constraint.

The richness of the present model framework, in which the age and number of children may affect household preferences, has previously precluded structural estimation of the effect of children on consumption. When the influential study by [Attanasio, Banks, Meghir and Weber \(1999\)](#) investigated the effect of children, it was an achievement to even simulate data from their model. Due to significant improvements of computers and computational techniques, I am able to formulate a similar model and estimate all structural parameters.⁵

The next section describes the conventional Euler equation estimators of the effect of children on consumption and show in a simple four-period model, with an analytical solution, how potentially binding credit constraints induce an omitted variable bias and use this bias to uncover bounds of the effect of children on consumption. Section 3 formulates a life cycle model of household consumption in the presence of children and credit constraints and Section 4 confirms the results from the four-period model through a Monte Carlo study. Section 5 provides an alternative structural estimation approach to uncovering *all* structural parameters including the effect of children on consumption. Section 6 apply the structural estimation approach and report estimated effects of children on consumption using both Danish register data and the PSID along with estimated bounds using only the Euler equation. Finally, Section 7 concludes with a discussion.

2 Euler Equation Estimation of the Effect of Children

All existing studies, to the best of my knowledge, analyzing the effect of children on household consumption, apply estimators based on the consumption Euler equation. The “standard” Euler equation does, however, not hold if households are potentially credit constrained. This is well known and [Adda and Cooper \(2003\)](#) show how Euler equation estimation of the intertemporal elasticity of substitution is upwards biased if credit constraints are ignored. I show that ignored credit constraints also produce inconsistent estimates of the effect of children on consumption when conventional Euler equation estimation techniques are applied. Further, I exploit this inconsistency to estimate bounds of the effect of children.

⁵Specifically, I utilize the Endogenous Grid Method (EGM), proposed by [Carroll \(2006\)](#). This method solves consumption models, as the one in the present study, extremely fast as documented in [Jørgensen \(2013\)](#).

Following [Rendahl \(forthcoming\)](#), the *constrained* Euler equation is

$$\begin{aligned}
u'(C_t)v(\mathbf{z}_t; \theta) - \lambda_t &= R\beta\mathbb{E}_t[u'(C_{t+1})v(\mathbf{z}_{t+1}; \theta) - \lambda_{t+1}] \\
&\Downarrow \\
R\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{v(\mathbf{z}_{t+1}; \theta)}{v(\mathbf{z}_t; \theta)} &= \underbrace{\epsilon_{1,t+1} + \epsilon_{2,t+1}}_{\equiv \epsilon_{t+1}},
\end{aligned} \tag{1}$$

where $\mathbb{E}_t[\cdot]$ denotes expectations conditional on information available in period t , λ_s is the shadow price of resources in period s , R is the real gross interest rate, β is the discount factor, C_t denotes consumption, $u(\cdot)$ is the utility function, assumed to be constant relative risk aversion (CRRA) with risk aversion parameter, ρ . $v(\mathbf{z}_{t+1}; \theta)$ is a taste shifter in which \mathbf{z}_t contains variables describing household demographics and θ is their loadings such that θ is the parameter vector that smooths marginal utility across periods with changing household composition. Unless otherwise stated, as is standard in the literature, $v(\mathbf{z}_t; \theta) = \exp(\theta \# \text{kids}_t)$. The structural Euler residual, ϵ_s , satisfy

$$\begin{aligned}
\mathbb{E}_t[\epsilon_{1,t+1}] &= 1, \\
\epsilon_{2,t+1} &= -\frac{\lambda_t - R\beta\mathbb{E}_t[\lambda_{t+1}]}{u'(C_t)v(\mathbf{z}_t; \theta)}.
\end{aligned}$$

From the Kuhn-Tucker conditions we know that $\lambda_t \geq 0$ and $\lambda_t(M_t - C_t + \kappa P_t) = 0$ in all time periods. Hence, if agents know with perfect certainty that the borrowing constraint will *not* bind in *any* period, we have that $\mathbb{E}_t[\epsilon_{t+1}] = 1$. Generally, however, consumers are *not* certain that they will be unaffected by constraints and the mean expectational error in (1) is a function of information today,

$$\mathbb{E}_t[\epsilon_{t+1}] = f(C_t, \mathbf{z}_t) \neq 1,$$

and serially correlated through the presence of λ_t and λ_{t+1} in $\epsilon_{2,t+1}$. In the existing literature on intertemporal consumption allocation and the effect of children on consumption, credit constraints are effectively ignored or assumed away.

Ignoring potentially binding credit constraints (i.e., assuming that $\lambda_s = 0 \forall s$), a non-linear GMM estimator of θ could be

$$\theta_{GMM} = \underset{\theta}{\operatorname{argmin}} \left[\frac{1}{NT} \sum_i^N \sum_t^T \left(R\beta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\rho} \exp(\theta \Delta \mathbf{z}_{i,t+1}) - 1 \right) \cdot Z_{i,t+1} \right]^2, \tag{2}$$

such that θ_{GMM} is the parameter that satisfy the sample equivalent of $\mathbb{E}[(\epsilon - 1)'Z] = 0$, where Z contain instrument(s) assumed uncorrelated with the Euler residual. Ignoring measurement error, the estimator in (2) produce consistent estimates as long as a suitable instrument is available and, importantly, households does in fact not face credit constraints. [Alan, Attanasio and Browning \(2009\)](#) supply modified GMM estimators to allow for measurement error while still ignoring possibly binding credit constraints. The results will transfer directly to more realistic settings in which consumption is observed with measurement error.

Using food consumption from the PSID, [Alan, Attanasio and Browning \(2009\)](#) estimate the

effect of children to be around .18 from a similar estimator as (2) and as large as .9 using estimators allowing for measurement error in consumption.⁶ [Dynan \(2000\)](#), also using the PSID and a GMM estimator, estimates almost as large effects of children of around $\theta \approx .7$.

Most existing studies work with a log-linearized version of the Euler equation resulting in the estimable equation

$$\Delta \log C_{it} = \text{constant} + \rho^{-1} \theta' \Delta \mathbf{z}_{it} + \tilde{\epsilon}_{it}, \quad (3)$$

in which the first term is a constant as a function of structural parameters (β, ρ) and the interest rate (assumed constant), the second term is the effect of children and the last term is a reduced form residual, $\tilde{\epsilon} = -\rho^{-1} \log \epsilon$. Instruments such as lagged changes in number of children or cohort-average number of children are typically used to circumvent endogeneity of the interest rate and children.

In the influential study by [Attanasio, Banks, Meghir and Weber \(1999\)](#), θ and ρ is uncovered by estimating a log-linearized Euler equation using lagged changes in \mathbf{z} as instruments along with lagged changes in income and consumption. The effect of the number of children is found to be around $\theta \approx .33$ using the CEX. Several studies have used the PSID to estimate versions of the log-linearized Euler equation, see, e.g., [Hall and Mishkin \(1982\)](#); [Runkle \(1991\)](#) and [Lawrance \(1991\)](#). The latter reports estimates suggesting a value of θ of around 0.5. [Browning and Ejrnæs \(2009\)](#) allow for a more flexible functional form of $v(\mathbf{z}_t; \theta)$ when estimating the effect of children on log consumption growth using the FES and argue that the number and age of children can explain completely the hump in consumption.

Some empirical studies recognize that credit constraints might affect household behavior. Often, potentially binding credit constraints are handled by discarding households in which nothing is carried over from period t to $t + 1$ (see, e.g., [Alan, Attanasio and Browning, 2009](#)). This strategy is clearly not a satisfactory approach because households expectations about the possibility of the credit constraint being binding in future periods ($\mathbb{E}_t[\lambda_{t+1}]$) still affect present consumption behavior. Determining at which level of wealth households are completely free of the credit constraint is non-trivial.

Other estimators have been proposed to estimate Euler equations. For example, [Alan and Browning \(2010\)](#) propose a method in which they fully parametrize the Euler residuals and simulate these residuals and consumption paths simultaneously. Their Synthetic Residual Estimation (SRE) procedure does, like most other methods, not allow for credit constraints in a coherent way. Since the GMM and log-linearized estimation methods are the conventional methods used in the literature, I focus exclusively on these. I conjecture, however, that all conclusions would carry directly to other alternative estimation methods, as SRE, that does not handle credit constraints and other potential life cycle motives in a structured way.

Estimation of log-linearized Euler equation has been discussed intensively in the literature. For example, [Carroll \(2001\)](#) argues that estimation of ρ using a log-linearized Euler equation suffers from an omitted variable bias if consumers face income uncertainty. [Attanasio and Low](#)

⁶They use the lagged changes in number of children, $\Delta \mathbf{z}_t$, as instrument and the estimates are very imprecise. Since the change in the number of children inside a given household is a very persistent series, the large and imprecise estimates might be due to the use of a weak instrument. I find in the Monte Carlo study below indications of a weak instrument problem using lagged changes in household-specific number of children. Using the change in birth cohort average number of children as instrument produce better results.

(2004) argue, however, in favor of Euler equation estimation arguing that the critique in Carroll (2001) is unwarranted in practice. I ignore any considerations on this discussion and assume, for the ease of exposition, that researchers *know* all other parameters except the effect of children on consumption, θ .

The present exposition of the inconsistency of previously applied estimators is, therefore, based on the absolutely best of all circumstances in which *i)* a panel of consumers is available, *ii)* consumption is observed without measurement error, *iii)* researchers know the underlying model consumers solve, and *iv)* researchers know the preferences of consumers except the effect of children on consumption. Even in these unrealistically good circumstances, I will show that ignoring potentially binding credit constraints will produce inconsistent estimates of the effect of children on household consumption using both the exact Euler equation (2) or the log-linearized Euler equation (3).

This has not been discussed or documented before and pose a serious problem when estimating the effect of children on consumption since the estimators described above does not easily generalize to the case when households face possibly binding credit constraints. Before turning to a full structural life cycle model of intertemporal consumption choices, similar to model frameworks used in existing literature, I formulate a tractable four-period life cycle model with an analytical solution. This model serves as a simple illustration of the incentives in the richer model and illustrates all results confirmed by the baseline model.

The results generalize to cases in which consumers do not face credit constraints. If there instead is some probability \wp of receiving a zero-income shock (as in Carroll, 1997 and Gourinchas and Parker, 2002), all results concerning the log-linearized Euler equation (3) still hold.⁷ This is basically because risk averse consumers will instead face a “self-imposed” no-borrowing constraint stemming from the fear of receiving zero income in all future periods with consumption of zero as a consequence. In turn, consumption will respond substantially to negative income shocks if either explicit or self-imposed credit constraints affect consumers, increasing the variance in consumption growth. Because higher order moments (such as something like the variance of consumption growth, Carroll, 2001) enters the reduced form residual, $\tilde{\epsilon}$, log-linearized Euler equation estimation will not be able to uncover the effect of children on consumption.⁸

2.1 An Illustrative Model of Consumption and Children over the Life Cycle

The model consists of four periods. In the initial period, period zero, no child is present in any household. In period one, the “young” stage, with probability p , a child arrives, $\mathbf{z}_1 = 1$, and with probability $1 - p$ the household remains childless, $\mathbf{z}_1 = 0$. In a deterministic version of the model, children arrive with certainty in p households while a share $1 - p$ remains childless. In the second period, the “old” stage, the child moves (if present in period one) such that $\mathbf{z}_2 = 0$ for all households. Households die with certainty in the end of period three and, since there is no bequest motive, consume all available resources in this terminal period.

Utility is CRRA and the taste shifter is assumed to be given by $v(\mathbf{z}_t; \theta) = \exp(\theta \# \text{kids}_t)$

⁷In absence of “explicit” credit constraints, the Euler equation in (1) has mean one because $\lambda_t = 0$, $\forall t$ and the exact GMM estimator produce unbiased estimates of the effect of children.

⁸I confirm this argument by Monte Carlo simulations in Section 4.

with baseline parameters of $\rho = 2$ and $\theta = 0.5$. Children are binary such that either a child is present or not. To reduce unnecessary cluttering in equations, the gross real interest rate and the discount factor both equal one, $R = \beta = 1$. Households receive a deterministic income of Y_t in beginning of every period. Income grow with G_1 between period zero and period one ($Y_1 = G_1 Y_0$) and is constant otherwise ($Y_t = Y_{t-1}$, $t = 2, 3$). The beginning-of-period resources available for consumption is the sum of household income and end-of-period wealth carried over from last period, $M_t = A_{t-1} + Y_t$.

Formally, when children arrive *probabilistically*, households solve the problem

$$\max_{C_0, C_1, C_2} \frac{C_0^{1-\rho}}{1-\rho} + p \exp(\theta) \frac{C_1^{1-\rho}}{1-\rho} + (1-p) \frac{C_1^{1-\rho}}{1-\rho} + \frac{C_2^{1-\rho}}{1-\rho} + \frac{(M_2 - C_2 + Y_3)^{1-\rho}}{1-\rho},$$

while households, when children arrive *deterministically*, solve the problem for a given value of $\mathbf{z}_1 \in \{0, 1\}$,

$$\max_{C_0, C_1, C_2} \frac{C_0^{1-\rho}}{1-\rho} + \exp(\theta \mathbf{z}_1) \frac{C_1^{1-\rho}}{1-\rho} + \frac{C_2^{1-\rho}}{1-\rho} + \frac{(M_2 - C_2 + Y_3)^{1-\rho}}{1-\rho},$$

both subject to a no-borrowing constraint, $A_t \geq 0$, $\forall t$. Appendix B solves the model analytically and reports the resulting optimal consumption functions. Note, the model in which children arrive probabilistically has no closed form solution for period-zero consumption and is solved numerically to complete arguments and present simulated consumption profiles.

Figure 1 presents consumption profiles for households initiated with no wealth in the initial period, $A_{-1} = 0$, early life income growth of eight percent, $G_1 = 1.08$ and a 50 percent share of households with children in period one, $p = 0.5$. Panel (a) presents the consumption profile from a model in which children arrive probabilistically while the deterministic version is presented in panel (b). The solid black line illustrates the consumption profile of households in which *no* child arrives in period one, the black dashed line represents households in which no child arrives in a model with credit constraints, the red line illustrates the consumption profile of a household in which a child (randomly) arrives in period one in absence of constraints, and the red dashed line represents the consumption profile of the latter household in the presence of a no-borrowing constraint. Clearly, potentially binding credit constraints affect the consumption profiles significantly. Consumption growth is in general lower when a child arrives in models with credit constraints.

The difference between the stochastic and deterministic arrival of children is the heterogeneity in consumption *prior* to the arrival of children. Consumption in period zero is lower in the stochastic model relative to consumption of households who do not have children in the deterministic model, $C_0^*(M_0) \leq C_0^*(M_0 | \mathbf{z}_1 = 0)^{\text{det}}$. This illustrates that, in the stochastic model, *all* households irrespectively of whether a child actually arrives in period one will accumulate wealth to buffer against the potential arrival of a child as shown in Figure 1a.

In the stochastic model, consumption of young households (period one) will increase more than income increases even if a child does *not* arrive. In period one it has been revealed to childless households that they will not have children and the wealth accumulated in previous period to buffer against the potential arrival of a child will be evenly distributed across remaining periods. Contrary, in the deterministic model, childless households will increase consumption exactly as much as income grows as shown in Figure 1b and is only affected by children if they

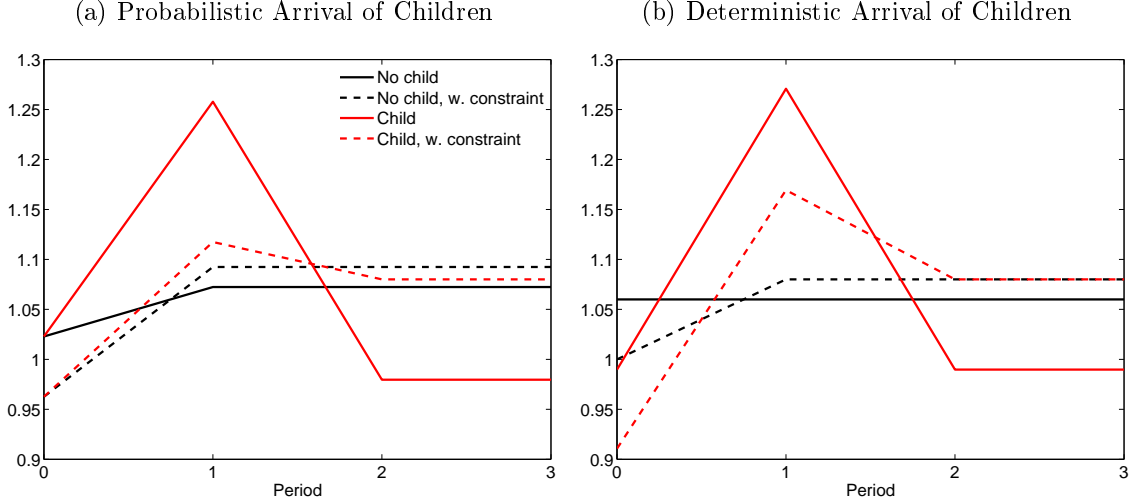


Figure 1 – Consumption Profiles from the Simple Four-period Model, $p = 0.5$, $\rho = 2$, $G_1 = 1.08$, and $\theta = 0.5$.

arrive. This, in effect, results in childless households, in the deterministic model, only being potentially credit constrained in period zero because they are unable to borrow against future income growth.

2.2 Inconsistency and Bounds from Euler Equation Estimation

Using the simple four-period model, I show analytically that *i*) Euler equation estimation produce inconsistent estimates of the effect of children on consumption when households face possibly binding credit constraints, *ii*) young households can be used to estimate an upper bound, and *iii*) older households can be utilized to estimate a lower bound. I confirm that these results hold in a more general life cycle model by Monte Carlo simulations in Section 4 below.

Inconsistency (Young Households). Consider first the model in which children arrives *deterministically*. Then, the Euler equation linking initial consumption, C_0 , with consumption of young households is

$$C_0(\mathbf{z}_1)^{-\rho} - \lambda_0(\mathbf{z}_1) = \exp(\theta \mathbf{z}_1) C_1^{-\rho} - \lambda_1(\mathbf{z}_1),$$

where $\lambda_0(\mathbf{z}_1)$ is the shadow price of resources in period zero and $\lambda_1(\mathbf{z}_1)$ is the shadow price of resources in period one, as increasing functions of \mathbf{z}_1 .⁹ As argued above, childless households, in the deterministic version, is potentially credit constrained only in period zero since they are not allowed to borrow against the future income growth implying that $\lambda_0(0) \geq 0$, $\lambda_1(0) = 0$. Also, the credit constraint has more bite in period one relative to period zero, $\lambda_0(1) \leq \lambda_1(1)$, if a child arrives. Comparing log consumption growth across households with and without children

⁹If children increase the marginal utility of consumption ($\theta > 0$) the credit constraint will be “more” binding if a child arrives compared to if no child arrives.

(OLS) yields a *negative bias*,

$$\Delta \log(C_1)^{\det}|_{\mathbf{z}_1=1} - \Delta \log(C_1)^{\det}|_{\mathbf{z}_1=0} = \rho^{-1}\theta - \rho^{-1} \log \left[\frac{1 - (\lambda_0(1) - \lambda_1(1))C_0(1)^\rho}{1 - \lambda_0(0)C_0(0)^\rho} \right] \leq \rho^{-1}\theta, \quad (4)$$

since the nominator is greater than one while the denominator is less than one. The OLS estimate for the baseline parameters is 0.17, significantly less than the true value of $\rho^{-1}\theta = .25$. Only if the credit constraint does *not* bind can the effect of children on consumption be uncovered if children arrive deterministically.

When children arrive *probabilistically*, the effect of children on consumption cannot successfully be uncovered even if credit constraints do not bind ($\lambda_0 = \lambda_1 = 0$) by comparing consumption growth across household with and without children. Recall that all households are in the initial period identical in the stochastic version and when, in period one, it is revealed whether a child has arrived, consumption increases even if a child does *not* arrive.

Comparing log consumption growth in the stochastic model across households with and without children confirms this (assuming $\lambda_0 = \lambda_1 = 0$),

$$\Delta \log(C_1)|_{\mathbf{z}_1=1} - \Delta \log(C_1)|_{\mathbf{z}_1=0} = \rho^{-1}\theta - (\log(\exp(\rho^{-1}\theta) + 2) - \log(3)) \leq \rho^{-1}\theta,$$

which equals $\rho^{-1}\theta$ if and only if $\theta = 0$ (no effect of children). For the parameters used here ($\rho = 2$ and $\theta = 0.5$), the Euler equation estimate is 0.16. This downwards bias is exacerbated if households face possibly binding credit constraints; for the parameters used here, the OLS estimate of the effect of children in the model with a credit constraint is 0.02.¹⁰

In sum, there is a *downwards bias* in the Euler equation estimate of the effect of children on consumption if households face credit constraints, using young households. Only if the credit constraint does not bind can the effect of children be successfully uncovered from the Euler equation of young households. Even in this case, if children arrive probabilistically, the estimate of the effect of children will be downwards biased.

Upwards Bias of IV Using Young Households in Which Children Arrive: An Upper Bound.

Combined with the credit constraint, if income grows faster in the early part of life, a *positive bias* may also distort estimation results. To see this, recall that income grows with G_1 between period zero and one and is constant across all other periods. For the ease of exposition, assume that households are credit constrained in the initial period (prior to children) and is also credit constrained in period one only if a child arrives. This scenario is relevant since it is a parsimonious description of young households: they are (at least) initially credit constrained and have a steep income path. Under these assumptions, households who have children will increase consumption with as much as income grows.

Using the cohort-average number of children as instrument, $Z = \Delta \bar{\mathbf{z}}_1 = p$, will produce an upwards bias for even moderate income growth. Imagine that there is no effect of children ($\theta = 0$) while maintaining the previous assumptions that households are credit constrained in

¹⁰As discussed in Appendix B, there is no closed form solution to the initial period consumption if children arrive probabilistically. The optimal consumption is, therefore, found numerically in this case.

the initial period. The IV estimate of the effect of children on consumption is then

$$\begin{aligned}
\frac{\mathbb{E}[\Delta \log C'_1 Z]}{\mathbb{E}[Z'Z]} &= \frac{1}{p}(p\Delta \log(C_1)|_{\mathbf{z}_1=1} + (1-p)\Delta \log(C_1)|_{\mathbf{z}_1=0}), \\
&= \frac{1}{p}(p\Delta \log(C_1) + (1-p)\Delta \log(C_1)), \\
&= \log(G_1)/p \geq \rho^{-1}\theta = 0,
\end{aligned} \tag{5}$$

such that the Euler equation IV estimation is upwards biased if households are initially constrained and income grows.

If the effect of children is “large enough”, households want to accumulate sufficient wealth to escape the credit constraint and ensure a situation in which they can increase consumption as desired, when children arrive. Hence, for large values of θ , households will in effect act as if there were no credit constraints. In such a situation, if children arrive probabilistically, using the cohort-average number of children will still produce a positive bias in the estimate of the effect of children, but the bias is *very* small.¹¹ To see this, imagine that children arrive probabilistically and there is no credit constraints. $\Delta \bar{\mathbf{z}}_1 = p$ is used as instrument such that after very tedious algebra the IV estimator is given by (see Appendix B.3 for the derivation)

$$\begin{aligned}
\frac{\mathbb{E}[\Delta \log C'_1 Z]}{\mathbb{E}[Z'Z]} &= \frac{1}{p}(p\Delta \log(C_1)|_{\mathbf{z}_1=1} + (1-p)\Delta \log(C_1)|_{\mathbf{z}_1=0}), \\
&= \rho^{-1}\theta + \omega \geq \rho^{-1}\theta,
\end{aligned} \tag{6}$$

where

$$\omega \equiv p^{-1} [\rho^{-1} \log(p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho) - (p \log(\exp(\rho^{-1}\theta) + 2) + (1-p) \log(3))].$$

The bias, ω , is the difference between the log-expected value, $\log \mathbb{E}$, and the expected-log value, $\mathbb{E} \log$.¹² Since the logarithm is a concave function the former is always greater than the latter ($\log \mathbb{E} > \mathbb{E} \log$) such that the bias is positive. Note, however, that this bias is rather small and vanishes asymptotically, such that $\lim_{\rho \rightarrow \infty} \omega = 0$ and $\lim_{p \rightarrow 1} \omega = 0$. For the parameters used here, $\rho = 2$, $\theta = .5$, and $p = .5$, the bias is only $\omega \approx 0.004$. This very small bias is also found in Monte Carlo simulations from the richer baseline model below.

This upwards bias motivates the use of young households to estimate an *upper bound* on the effect of children using the cohort-average number of children as instrument. In general, using the cohort-average number of children as instrument produce an upwards bias. If the effect of children is “large enough”, households will accumulate sufficient wealth prior to the arrival of children to escape the constraint, reducing the bias significantly. In this case, the IV estimate is *very* close to the true effect of children.¹³

¹¹If children arrive deterministically, the effect of children can be uncovered using the change in number of children, $\Delta \mathbf{z}_1$, as shown in (4)

¹²Defining $\omega_1 \equiv (\exp(\rho^{-1}\theta) + 2)^\rho$ and $\omega_2 \equiv 3^\rho$, the error is $\omega = p^{-1}\rho^{-1}(\log(p\omega_1 + (1-p)\omega_2) - (p \log \omega_1 + (1-p) \log \omega_2))$.

¹³The use of an instrument, when credit constraints are not binding, is only required if children arrive probabilistically. If children are perfectly foreseen, as in the deterministic model, the change in the number of children can identify the effect of children on consumption, as shown in equation (4).

Downwards Bias Using Older Households in Which Children Leave: A Lower Bound.

I now analyze the identification of θ using only older households from which children move with certainty in period two. The Euler equation linking period one and period two consumption is given by

$$C_1^{-\rho} \exp(\theta \mathbf{z}_1) - \lambda_1(\mathbf{z}_1) = C_2^{-\rho} - \lambda_2(\mathbf{z}_1),$$

such that comparing log consumption growth (negative since children move) across households with and without children yields a *negative bias*,

$$-(\Delta \log(C_2)|_{\mathbf{z}_1=1} - \Delta \log(C_2)|_{\mathbf{z}_1=0}) = \rho^{-1}\theta + \rho^{-1} \log \left[\frac{1 - (\lambda_1(1) - \lambda_2(1))C(1)_1^\rho \exp(-\theta)}{1 - (\lambda_1(0) - \lambda_2(0))C(0)_1^\rho} \right] \leq \rho^{-1}\theta.$$

This is because if the childless household was credit constrained in period one ($\lambda_1(0) > 0$), they will also be constrained in period two implying that the household with children also will be credit constrained in both periods (since $\theta > 0$). In turn, consumption growth across households with and without children are identical and the OLS estimate of the effect of children will be zero. Unless there is no effect of children ($\theta = 0$), this is *less* than the true effect. On the other hand, if childless households are *not* constrained in period one, they will also not be constrained in period two ($\lambda_1(0) = \lambda_2(0) = 0$) producing a negative bias since $\lambda_1(1) \geq \lambda_2(1) \Rightarrow \log[\cdot] \leq 0$, when $\theta \geq 0$.

In conclusion, there is a negative bias from Euler equation estimation of the effect of children on consumption, using older households. For the parameters used here ($\rho = 2$ and $\theta = 0.5$), the Euler equation estimate is 0.13 and 0.11 for the deterministic and stochastic models, respectively.¹⁴ Only if the credit constraint is not binding in either period, can the effect of children be uncovered from the Euler equation of older households.

This downwards bias can be utilized to estimate a *lower bound* of the effect of children on consumption. If the effect of children is relatively *small*, the credit constraint is likely to be less binding for older households and the effect of children can be identified from older households. Otherwise, if the effect of children is large, the OLS estimate using older households will be less than the actual effect of children.

Table 1 summarizes the Euler equation estimation results from the four-period model. Under estimation is indicated with an underline while an overline highlight over estimation. It is clear that OLS using older households produce a lower bound and if credit constraints are absent uncover the underlying effect of children on consumption. Independent of whether children arrive deterministically or probabilistically, IV estimation using only younger households produce an upper bound. If children arrive deterministically the true effect of children is uncovered using the change in the cohort-average number of children.

¹⁴The bias is larger for stochastic model since households smooth *expected* utility and when a child subsequently arrives the level of wealth accumulated is less than what is accumulated in the deterministic model. Hence, the constraint will be more severe in the stochastic model increasing λ_1 .

Table 1 – Euler Equation Estimation Results, Four-Period Model, $\rho^{-1}\theta = 0.25$.

Arrival of children	Young		Older	
	OLS	IV	OLS	IV
<i>Unconstrained</i>				
Probabilistically	<u>0.160</u>	<u>0.254</u>	0.250	0.250
Deterministically	0.250	0.250	0.250	0.250
<i>Constrained</i>				
Probabilistically	<u>0.023</u>	<u>0.276</u>	<u>0.034</u>	<u>0.034</u>
Deterministically	<u>0.173</u>	<u>0.327</u>	<u>0.079</u>	<u>0.079</u>

Notes: Log-linearized Euler equation results of $\widehat{\rho^{-1}\theta}$ from four-period model. Parameters are $\beta = 1$, $\gamma = 1$, $\rho = 2$, $\theta = 0.5$, $p = 0.5$, and $G_1 = 1.08$. An overline marks over estimation (compared to $\rho^{-1}\theta = 0.25$) and an underline marks under estimation.

3 The Baseline Model

The framework used throughout the rest of this study is a version of the buffer-stock model which has been used extensively for different purposes in the consumption literature. The model captures the main consumption and savings incentives of households over the life cycle prior to retirement. Specifically, the model is very similar to the one used in [Attanasio, Banks, Meghir and Weber \(1999\)](#); [Gourinchas and Parker \(2002\)](#) and [Cagetti \(2003\)](#). Similar models are used in many different areas of research in which household demographics often are ignored while other features are augmented.¹⁵

Households work until an exogenously given retirement age, T_r , and die with certainty at age T where they consume all available resources. In all preceding periods, households solve the optimization problem

$$\max_{C_t} \mathbb{E}_t \left[\sum_{\tau=t}^{T_r-1} \beta^{\tau-t} v(\mathbf{z}_t; \theta) u(C_\tau) + \gamma \sum_{s=T_r}^T \beta^{s-t} v(\mathbf{z}_t; \theta) u(C_s) \right], \quad (7)$$

where utility is CRRA and $v(\mathbf{z}_t; \theta)$ is a taste shifter, monotonically increasing in \mathbf{z}_t , which contains variables describing the number and age of children and θ is their loadings.

Following [Gourinchas and Parker \(2002\)](#), survival and income uncertainty are omitted post retirement and the parameter γ (referred to as the retirement motive) in equation (7) is a parsimonious way of adjusting for these elements.¹⁶ [Gourinchas and Parker \(2002\)](#) ignore the post-retirement consumption decisions and adjust the perfect foresight approximation by a parameter similar to γ through a retirement value function. Although I focus on consumption behavior *prior* to retirement, the potential presence of children at retirement forces the model

¹⁵Some examples (with a sample of references) are estimation of the rate of time preference ([Lawrance, 1991](#); [Dynan, 2000](#); [Alan and Browning, 2010](#)) housing decisions ([Yang, 2009](#); [Marekwica, Schaefer and Sebastian, 2013](#)), consumption inequality and partial insurance ([Storesletten, Telmer and Yaron, 2004](#); [Blundell, Pistaferri and Preston, 2008](#); [Heathcote, Storesletten and Violante, forthcoming](#)), and retirement choices ([Blau, 2008](#); [van der Klaauw and Wolpin, 2008](#)).

¹⁶Survival is also certain prior to retirement.

to be specific about post retirement behavior as well. Specifically, ignoring the presence of children after retirement (as done in [Gourinchas and Parker, 2002](#)) would lead the model to under estimate optimal consumption in periods prior to retirement.¹⁷ Since the focus of this study is on estimation of θ , it is essential that aspects related to children are properly handled.

Households solve (7) subject to the intertemporal budget constraint,

$$M_{t+1} = R(M_t - C_t) + Y_{t+1},$$

where R is the gross real interest rate, M_t is resources available for consumption in beginning of period t and Y_t is beginning-of-period income. While working, income is assumed to follow the stochastic process

$$\begin{aligned} Y_t &= P_t \varepsilon_t, \forall t < T_r, \\ P_t &= G_t P_{t-1} \eta_t, \forall t < T_r, \end{aligned}$$

where P_t denotes permanent income, G_t is real gross income growth, $\eta_t \sim \log \mathcal{N}(-\sigma_\eta^2/2, \sigma_\eta^2)$ is a mean one permanent income shock, and ε_t is a mean one transitory income shock taking the value μ with probability \wp and otherwise distributed $(1 - \wp)\varepsilon_t \sim \log \mathcal{N}(-\sigma_\varepsilon^2/2 - \mu\wp, \sigma_\varepsilon^2)$. When retired, the income process is a deterministic fraction $\varkappa \leq 1$ of permanent income and permanent income grows with a constant rate of G_{ret} once retired,

$$\begin{aligned} Y_t &= \varkappa P_t, \forall t \geq T_r, \\ P_t &= G_{ret} P_{t-1}, \forall t \geq T_r. \end{aligned}$$

In each period, households face an intratemporal budget,

$$\begin{aligned} M_t &= A_t + C_t, \\ A_t &\geq -\kappa P_t \forall t, \kappa \geq 0, \end{aligned}$$

such that end-of-period wealth, A_t , and consumption must equal the available resources in the beginning of the period and net wealth must be greater than a fraction $-\kappa$ of permanent income in all time periods. When retired, households are not allowed to be net borrowers, $A_t \geq 0, \forall t \geq T_r$, following [Gourinchas and Parker \(2002\)](#).

3.1 Household Composition

The evolution of children, \mathbf{z}_t , is normally ignored since household compositional effects typically are ignored or collapsed into a deterministic correction of the discount factor, identical for all households ([Attanasio, Banks, Meghir and Weber, 1999](#); [Gourinchas and Parker, 2002](#); [Cagetti, 2003](#); [Bick and Choi, 2013](#)). Such studies calculate an average age profile for the taste shifter (based on Euler equation estimation results) which *all* households irrespectively of the specific

¹⁷This is because too much is saved in the last working period if the decrease in marginal value of consumption in the future, when a child moves, is ignored. Households who know that, while they are retired, a child will move, will incorporate the associated drop in household consumption already before retiring since less wealth is required to maintain a given level of consumption while retired.

composition within a given household is assigned. This strategy would, however, not allow identification of the effect of children on household consumption because all households are affected identically. Since θ is of primary interest in this study, I will be precise about the underlying process regarding the arrival and leaving of children.

As most of the existing literature, I follow [Attanasio, Banks, Meghir and Weber \(1999\)](#) and let children affect the *marginal value* of consumption through a multiplicative $v(\mathbf{z}_t; \theta)$. Alternatively, the household composition could be included as a scaling of resources and consumption (equivalence scaling), as done in, e.g., [Fernández-Villaverde and Krueger \(2007\)](#). See [Bick and Choi \(2013\)](#) for an analysis of different approaches to and implied behavior from inclusion of household demographics in life cycle models. Alternative parametrizations would simply require reformulating the estimable equations accordingly and, thus, this particular parametrization is not generating any of the results.

Individuals do not die, divorce or remarry such that households consist of the same husband and wife at all times. Households can have at most three children and no infants arrive after the wife turns 43 years old. For notational simplicity, the age of each child is contained in \mathbf{z}_t ,

$$\mathbf{z}_t = (\text{age of child } 1_t, \text{ age of child } 2_t, \text{ age of child } 3_t) \in \{\text{NC}, [0, 20]\}^3,$$

where “NC” refers to “No Child” and the oldest child is denoted child one, the second oldest child as child two and the third oldest child as child three.

When a child is aged 21 the child does not influence household consumption in subsequent periods regardless of the value of θ . The arrival of an infant is stochastic with a known probability distribution depending on the age of the wife and number of children already present in the household. Households chose optimal consumption based on their (rational) expectations about arrival of children in future periods. Children could arrive deterministically while allowing for heterogeneity across households. If, e.g., all households know with perfect foresight how many children they will have and when these children arrive, a deterministic version of the model used throughout can be formulated.¹⁸

In the baseline stochastic case, not only households who have children are affected by the parameter θ . Households dynamically optimize their consumption behavior while incorporating expectations about the future such that *all* younger households within the same age group who have no children will want to reduce their consumption today in anticipation of increased consumption when children might arrive in the future. In the deterministic case, the savings rate will differ across households within age groups due to differences in when and how many children arrive over the life cycle. It is not obvious which is the most appropriate assumption (probabilistic or deterministic arrival on children) and the stochastic version has been chosen as baseline since that model does not require knowledge on completed fertility of individual households, when estimating model parameters. Results do differ significantly if children arrive deterministically rather than probabilistically, as indicated in the four-period model above and supported by Monte Carlo results below.

¹⁸Alternatively, and in between these two extremes, the choice of children could be endogenous in the model. For computational tractability I do not pursue that approach here but the very similar empirical results in Section 6.4 for both Danish and US consumers suggest that the arrival process of children does not drive the results.

4 Euler Equation Estimation: Inconsistency and Bounds

In this section, I confirm the results from the four-period model in Section 2 using the baseline life cycle model described in Section 3, often used in the literature: *i)* Euler equation estimation suffers from an omitted variable bias when households face potentially binding credit constraints, *ii)* When income growth and credit constraints interact, Euler equation estimation using only young households produce an upper bound of the effect of children, and *iii)* Using only older households produce a lower bound of the effect of children.

4.1 Inconsistency of Euler Equation Estimation: A Monte Carlo Study

Unlike the simple four-period model, the baseline model does not have an analytical solution. To simulate synthetic data, I solve the life cycle model using the Endogenous Grid Method (EGM) proposed by [Carroll \(2006\)](#) with “standard” parameters presented in Table 2. The technical details of the solution method are provided in Appendix C. The solution is then used to generate data for a given age and level of resources for 50,000 households from age 22 to 59. All households are initiated at age 22 with zero wealth, $A_{21} = 0$, permanent income of one (normalization), $P_{22} = 1$, and no previous children, $\mathbf{z}_{21} = (\text{NC}, \text{NC}, \text{NC})$.

Table 2 – Parameter Values Used to Simulate Data.

G_t	R	σ_ε^2	σ_η^2	κ	\wp	μ	β	ρ	γ	\varkappa	G_{ret}	θ
Fig. A.1	1.03	.005	.005	0	0	0	.95	2	1.1	.8	1.0	$\{0, 0.1, 0.3, 0.5\}$

In each period, a transitory and permanent income shock is randomly drawn from their respective distributions, described in Section 3, and multiplied permanent income to generate household income, Y_t . Last periods optimal end-of-period wealth is used together with household income to calculate beginning-of-period available resources, $M_t = RA_{t-1} + Y_t$. Figure A.1 presents the income growth rates. The income profile is concave and constant from age 40 to mimic empirical income profiles observed in most data.

To simulate the number and age of children in a household in a given period, the infant arrival probability is calculated using the PSID. The estimated arrival probabilities, as a function of age and number of children already present, are presented in the Figure A.2a in Appendix A. At each age, a uniform random variable is drawn and if the uniform draw is less than the arrival probability, a child arrives. When a child is 21 it moves and does not affect household behavior. The resulting profile of average number of children is presented in Figure A.2b and resembles the empirical profile from the PSID.¹⁹

Figure 2 presents simulated age profiles for income, consumption and wealth. To show how increasing the effect of children affects consumption and wealth accumulation over the life cycle, age profiles are shown for three different values of θ . All consumption profiles (even if children does not affect consumption) exhibit a hump when households are in the mid 40s, as typically observed in real data. If children affect consumption, the hump is exacerbated producing a

¹⁹The sharp kink in the average number of children after age 42 stems from the fact that all households are initiated with no children at age 21 such that at age 43 the children arrived at age 22 moves.

steeper consumption profile for young households and a subsequent larger decrease in consumption after the mid 40s. Income uncertainty, income growth and credit constraints produce an increasing consumption profile early in life, even if children does not affect consumption. This is a key feature of the model. If income is constant and certain and infinite borrowing is allowed, the consumption profile would be flat for childless households and only children can produce the hump in consumption.

The retirement motive, effectively reducing the value of consumption later in the working life, produces a downward sloping consumption profile after the mid 40s. Income is, post retirement, a deterministic fraction of permanent income and no borrowing is allowed once households are retired. This produce a strong incentive (depending on the size of γ) to accumulate wealth for retirement. Because households consume most of their income early in life (when income growth is high) the level of wealth accumulated around age 45 is not enough to smooth utility across retirement. Therefore, households reduce consumption later in life (when income grows less). In absence of this retirement motive, the consumption profile later in life would be flat. In combination with income growth and credit constraints, the consumption profile is hump shaped even if children does not affect consumption.

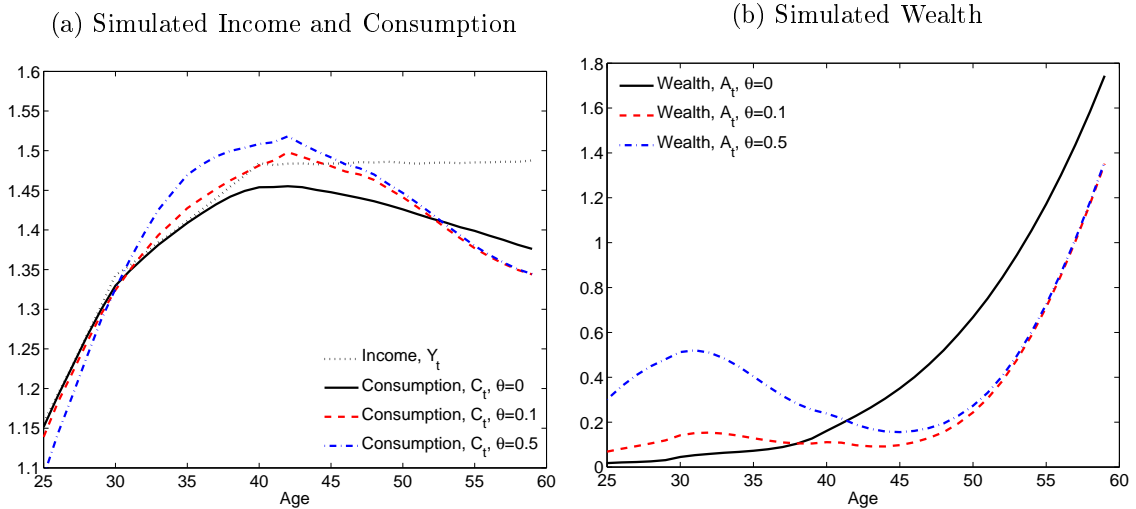


Figure 2 – Simulated Income, Consumption and Wealth Age Profiles. 50,000 households.

Interestingly, the consumption profiles across θ values are very similar for young households. This stems from income uncertainty and credit constraints preventing households from increasing consumption when children arrive despite they would want to, had the credit constraint not been present. Hence, the effect of children would be under estimated using young households, as shown earlier. If the effect of children is rather large, young households increase wealth accumulation in anticipation of children arriving in the future. When children subsequently arrive, wealth is significantly decumulated such that the credit constraint is binding for many households when children eventually move. The relative drop in consumption from a *constrained* level to an unconstrained level in general will be less than the relative change if households had never been constrained. Hence, the effect of children would be underestimated when only using

older households. This is exactly the point made using the simple four-period model.

When the effect of children is small ($\theta \leq 0.1$ here), the Euler residual will be positively correlated with the number of children. This is because consumption growth, even when children does not affect consumption, tend to be high for young households due to the credit constraint while the number of children also grows in this part of the life cycle. This positive correlation induce a positive bias in the estimation of the effect of children. Whether the negative or positive bias dominates is an empirical question.

In sum, the presence of credit constraints would tend to over estimate the effect of children on household consumption when the effect of children is small and under estimate the effect when it is large. I confirm this argument by applying the exact GMM estimator and the log-linearized Euler equation method to 1,000 independent simulated data sets for different values of θ_0 . Table 3 reports full-sample mean estimation results (and standard deviation across MC runs) using as instrument the actual change in number of children $\Delta \mathbf{z}_{t-s}$ for lags $s = 0, 1, 2$ and the change in the cohort-average number of children, $\Delta \bar{\mathbf{z}}_t$. To evaluate the performance for different lengths in the time dimension, 5 and 20 periods are used for estimation. All households are simulated from 22 through 59 and an age-window for each household is chosen randomly to start between age 25 and 54 (between 25 and 39 when $T = 20$ periods are used). The results are based on *all* households, as is standard in the literature, to show how the estimates would differ from the correct effect of children on consumption.

None of the estimation methods and instruments are able to uncover the true value of θ . For low levels of θ , there is a positive bias while for higher levels the bias is negative confirming the intuitive reasoning from the simple four-period model. The estimates are almost unaffected by the true value of θ and the bias persists as sample size increases. The large dispersion of parameters using lagged household-level instruments, $\Delta \mathbf{z}_{t-1}$ and $\Delta \mathbf{z}_{t-2}$ indicate that the number of children inside a household is so persistent that these instruments carry little information. Using the growth in cohort-average number of children produce similar point estimates but with greater precision.

To better understand and interpret the Monte Carlo results, Figure 3 plots the Euler residuals for different values of θ . The reduced form Euler residual, $\tilde{\epsilon}$, is shown in panel b. When the effect of children is low, the Euler residual is positively correlated with the growth in number of children, since $\tilde{\epsilon}$ decrease over the life cycle as the growth in number of children also tend to. This positive correlation produce a significant over estimation across all methods and instruments when $\theta = 0.0$.

When the effect of children are higher ($\theta > .1$ here) estimates are significantly lower than the true parameter for all sample sizes, estimation approaches and instruments. This downward bias is a result of the Euler residuals being negatively correlated with the number of children as shown in Figure 3 stemming from the fact that when the marginal value of consumption increase (decrease) significantly when children are present (move) the likelihood of the credit constraint binding in the future drops significantly when children move. Hence, the term $\lambda_t - R\beta\mathbb{E}[\lambda_{t+1}]$ increases producing a negative correlation between the growth in number of children and the error term.

Table 3 – Monte Carlo Results.

Instrument	$\theta = 0.0$		$\theta = 0.1$		$\theta = 0.3$		$\theta = 0.5$	
	LogLin	GMM	LogLin	GMM	LogLin	GMM	LogLin	GMM
$N = 1000, T = 5$								
$\Delta \mathbf{z}_t$	0.020 (0.008)	0.012 (0.008)	0.073 (0.009)	0.066 (0.009)	0.122 (0.010)	0.123 (0.010)	0.160 (0.011)	0.162 (0.011)
$\Delta \mathbf{z}_{t-1}$	0.218 (0.115)	0.124 (0.103)	0.199 (0.114)	0.122 (0.099)	0.107 (0.116)	0.073 (0.099)	0.121 (0.114)	0.090 (0.100)
$\Delta \mathbf{z}_{t-2}$	0.245 (0.189)	0.146 (0.163)	0.254 (0.185)	0.167 (0.157)	0.218 (0.178)	0.161 (0.149)	0.233 (0.173)	0.184 (0.149)
$\Delta \bar{\mathbf{z}}_t$	0.133 (0.021)	0.073 (0.021)	0.164 (0.021)	0.111 (0.020)	0.185 (0.022)	0.150 (0.021)	0.238 (0.023)	0.200 (0.024)
$N = 1000, T = 20$								
$\Delta \mathbf{z}_t$	0.015 (0.004)	0.006 (0.004)	0.073 (0.004)	0.065 (0.004)	0.119 (0.005)	0.118 (0.005)	0.144 (0.005)	0.145 (0.005)
$\Delta \mathbf{z}_{t-1}$	0.161 (0.044)	0.049 (0.031)	0.137 (0.046)	0.048 (0.032)	0.026 (0.050)	0.001 (0.034)	0.016 (0.051)	0.004 (0.034)
$\Delta \mathbf{z}_{t-2}$	0.177 (0.057)	0.052 (0.037)	0.194 (0.057)	0.077 (0.036)	0.151 (0.059)	0.075 (0.037)	0.143 (0.059)	0.084 (0.038)
$\Delta \bar{\mathbf{z}}_t$	0.113 (0.010)	0.035 (0.010)	0.146 (0.011)	0.072 (0.009)	0.161 (0.011)	0.103 (0.010)	0.198 (0.011)	0.138 (0.011)
$N = 50000, T = 5$								
$\Delta \mathbf{z}_t$	0.021 (0.001)	0.012 (0.001)	0.073 (0.001)	0.067 (0.001)	0.122 (0.001)	0.123 (0.001)	0.160 (0.002)	0.162 (0.002)
$\Delta \mathbf{z}_{t-1}$	0.210 (0.015)	0.120 (0.013)	0.193 (0.015)	0.119 (0.013)	0.108 (0.015)	0.073 (0.013)	0.123 (0.015)	0.090 (0.013)
$\Delta \mathbf{z}_{t-2}$	0.230 (0.022)	0.136 (0.019)	0.243 (0.022)	0.158 (0.019)	0.215 (0.022)	0.159 (0.018)	0.231 (0.021)	0.182 (0.019)
$\Delta \bar{\mathbf{z}}_t$	0.162 (0.003)	0.089 (0.003)	0.187 (0.003)	0.123 (0.003)	0.202 (0.003)	0.157 (0.003)	0.258 (0.004)	0.211 (0.004)
$N = 50000, T = 20$								
$\Delta \mathbf{z}_t$	0.015 (0.001)	0.006 (0.001)	0.073 (0.001)	0.065 (0.001)	0.119 (0.001)	0.118 (0.001)	0.144 (0.001)	0.145 (0.001)
$\Delta \mathbf{z}_{t-1}$	0.158 (0.006)	0.047 (0.005)	0.135 (0.007)	0.047 (0.005)	0.025 (0.007)	0.000 (0.005)	0.016 (0.007)	0.003 (0.005)
$\Delta \mathbf{z}_{t-2}$	0.178 (0.008)	0.054 (0.005)	0.196 (0.008)	0.079 (0.005)	0.154 (0.009)	0.078 (0.005)	0.147 (0.009)	0.087 (0.006)
$\Delta \bar{\mathbf{z}}_t$	0.125 (0.002)	0.038 (0.001)	0.155 (0.002)	0.073 (0.001)	0.166 (0.002)	0.102 (0.001)	0.204 (0.002)	0.137 (0.002)

Notes: Means and standard deviations (in parenthesis) are reported. Standard deviations are across monte carlo estimations. All results are based on 1,000 independent estimations on simulated data from the model described in Section 3 with the parameters presented in Table 2. The reported results for "LogLin" are of the "raw" estimate times ρ to facilitate a structural interpretation of the estimates. For each run, data are simulated for all N households from age 22 through 59 and a random adjacent period of length T is drawn from this simulation. All individuals are initiated at age 22 with zero wealth, $A_{21} = 0$, permanent income of one, $P_{22} = 1$, and no children. Children are assigned randomly following the estimated arrival probabilities estimated from the PSID, reported in Figure A.2.

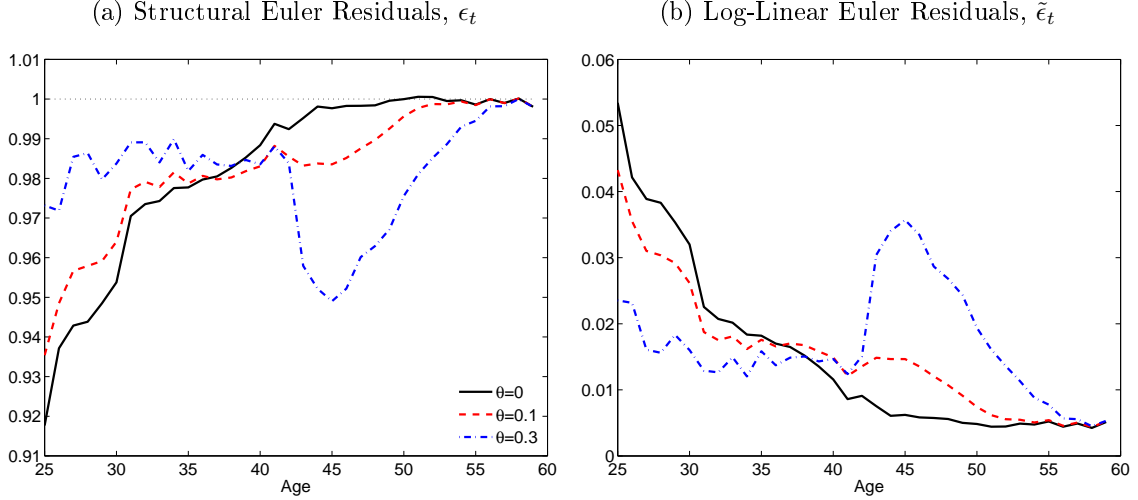


Figure 3 – Simulated Euler Residuals.

This illustrates the importance of household's expectations about the severity of the credit constraint, $\mathbb{E}[\lambda_{t+1}]$. As discussed earlier, existing studies often handle potentially binding credit constraints by discarding observations where $\lambda_t = 0$, i.e., end-of-period wealth is positive. This procedure ignores, however, the expectations about λ_{t+1} , something that can have devastating important implications for the consumption behavior in period t , as illustrated in Figure 3.

As argued in Section 2, the bias of the log-linearized Euler equation persists while GMM estimation is unbiased if there is no credit constraint but instead households face some small probability, $\wp > 0$, of becoming unemployed and receiving zero income, $\mu = 0$. To illustrate this, Figure A.3 in Appendix A reports the average Euler residuals from a model in which $\wp = 0.003$ and $\mu = 0$, following [Gourinchas and Parker \(2002\)](#). As expected, the structural Euler residual, ϵ , in this case vary randomly around a mean of 1 while the age profile of the reduced form log-linearized Euler residual is very similar to the one presented in Figure 3b from a model with an “explicit” credit constraint. This result extends the critique in [Carroll \(2001\)](#) on the inability of Euler equation estimation to uncover the intertemporal elasticity of substitution (ρ^{-1}) to the effect of children on consumption, θ .

4.2 Estimating Bounds of the Effect of Children Using the Euler Equation

4.2.1 Using older households and $\Delta \mathbf{z}_t$ to estimate a lower bound

As argued above and using the four-period model, older households will often not be credit constrained when children move but might be while they are present. In that case, $\text{cov}(\Delta \mathbf{z}_t, \tilde{\epsilon}_t) \leq 0$ producing a *negative bias* in the estimate of θ when using $\Delta \mathbf{z}_t$ as instrument. When the effect of children is sufficiently low, the likelihood that older households will *not* be credit constrained while children are present increases. This is because the consumption motive in the presence of children (in previous periods) has been less strong compared to the retirement motive and more wealth has been accumulated. This in turn results in $\lambda_t = \mathbb{E}[\lambda_{t+1}] = 0$ such that *no* bias is present and the true effect of children can be estimated.

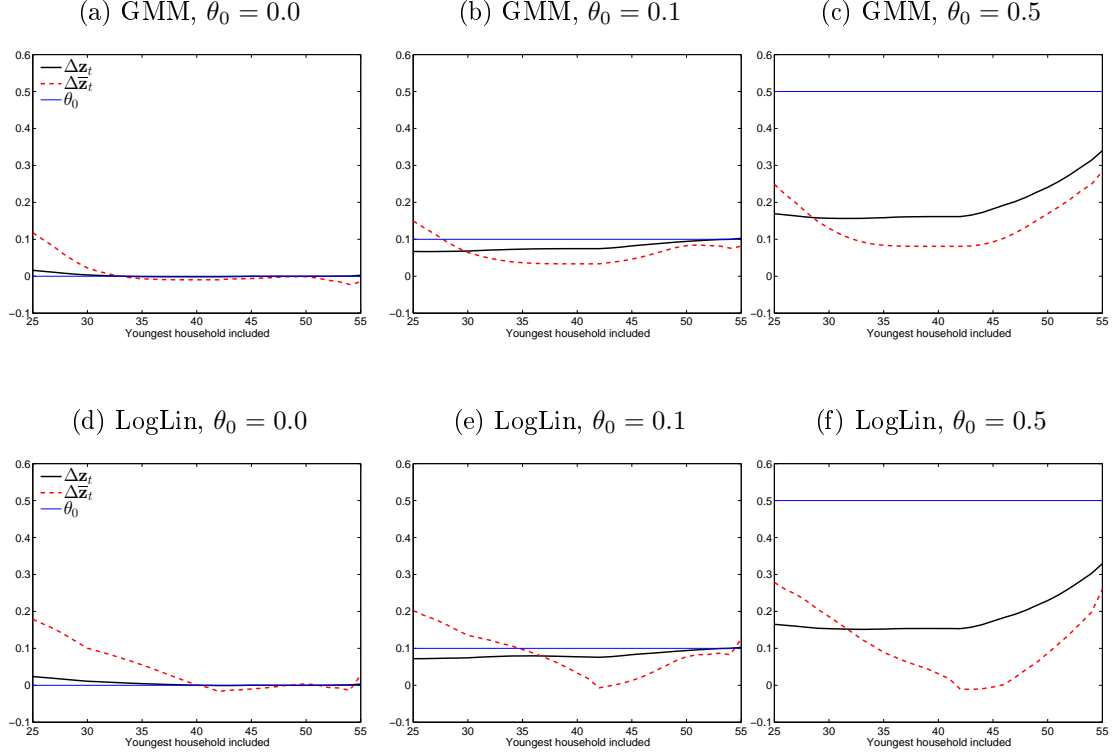


Figure 4 – Estimation on Sub-samples based on Age of Wife, from age 59 through 25.

To confirm this intuitive reasoning, Figure 4 presents results from GMM and log-linearized Euler equation estimation using 50,000 simulated households. Data are simulated as described above. The figure reports estimation results using different instruments to estimate $\theta \in \{0.0, 0.1, 0.5\}$ on sub-samples. The sub-samples are constructed such that increasingly younger households are included from age 59 through 25. For example, when the age of the youngest household included is 35, the estimation sample consists only of 35-59 year old households.

When the effect of children is sufficiently large, the Euler equation estimates using only older households are lower than the true value while for low values of θ , using only older households can uncover the true value of θ , confirming the argument above. Hence, using the actual change in number of children, $\Delta \mathbf{z}_t$, as instrument and restricting the estimation sample to only include older households produce a *lower bound* of θ . For completeness, the same exercise has been performed in a deterministic version of the model in which households have perfect foresight regarding arrival of children. The results are identical, as shown in Figure A.5 in Appendix A.

4.2.2 Using young households and $\Delta \bar{\mathbf{z}}_t$ as instrument to estimate upper bound

If the true value of θ is “too low” to induce consumers to accumulate wealth early in life, the estimation on young households using $\Delta \bar{\mathbf{z}}_t$ as instrument will produce estimates *above* the true parameter. If the true value of θ is “large enough” such that young households engage in accumulation of wealth before children arrive, an estimate very close to the true parameter can be uncovered using only young households and $\Delta \bar{\mathbf{z}}_t$ as instrument producing an *upper bound* of

the effect of children. This result was shown using the four-period model above.

Imagine a situation in which there is no effect of children on consumption ($\theta = 0$). When young, consumption will tend to grow (due to income growth and credit constraints) while the cohort-average number of children (\bar{z}_t) increases producing a *positive bias* in the estimation of θ using $\Delta\bar{z}_t$ as instrument. Imagine instead that children have a large effect on household consumption such that households will accumulate sufficient wealth to be able to increase consumption when children arrive. In this case, the credit constraint will have less bite ($\lambda_t = \mathbb{E}[\lambda_{t+1}] = 0$) and the effect of children can (almost) be uncovered. As discussed in relation to equation (6) when analyzing the four-period model, if children arrive probabilistically using $\Delta\bar{z}_t$ as instrument produces a *very small* positive bias when the effect of children is large if credit constraints does not affect consumers. In sum, using only young households and $\Delta\bar{z}_t$ as instrument producing an *upper bound* of the effect of children.

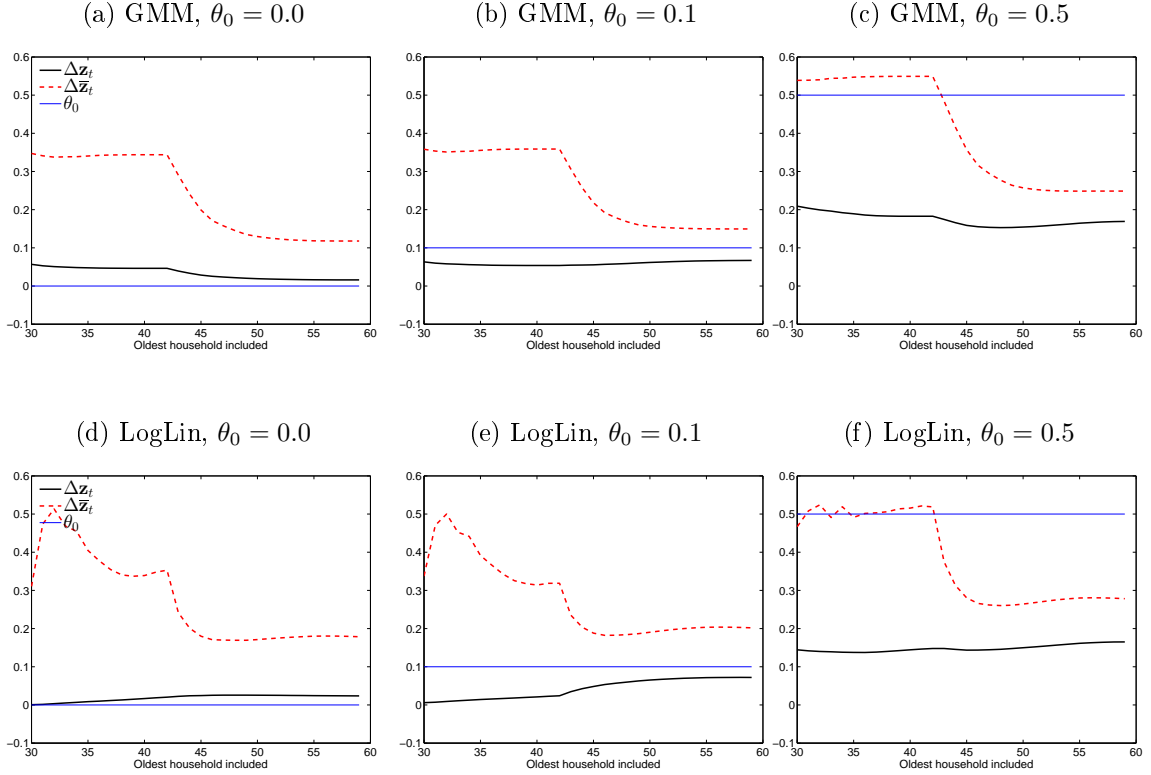


Figure 5 – Estimation on Sub-samples based on Age of Wife, from age 25 through 59.

To confirm this intuitive reasoning, Figure 5 presents results from GMM and log-linearized estimation using 50,000 simulated households. The figure reports estimation results using different instruments to estimate $\theta \in \{0.0, 0.1, 0.5\}$ on sub-samples. The sub-samples are constructed such that increasingly older households are included from age 25 through 59. For example, when the age of the oldest household included is 35, the estimation sample consists only of 25-35 year old households.

Clearly, using young households when θ is low produce over estimation. Interestingly, the estimate is *unaffected* by the true value of θ , illustrating identification issues when households

are potentially credit constrained. Using the simple four-period model, I showed in equation (5) that if households are credit constrained prior to the arrival of children and children have *no* effect on consumption, the proposed instrument produce an estimate of the effect of children proportional to log income growth. Hence, if the effect of children is small, the bias is large relative to the effect of children, and the IV estimate is almost independent of the true effect of children on consumption.

When the effect of children is large ($\theta = .5$ here) households accumulate a substantial amount of wealth early in life to be able to increase consumption when children arrive and the log-linear Euler equation produce something close to the correct parameter estimate, as predicted. The GMM estimator still over estimate the effect of children due to the mean Euler residual in (1) being less than one forcing the estimate of θ upwards in compensation. Since the bias is positive, GMM estimation can still be used as upper bound. Alternatively, using a additional moments will allow researchers to estimate the mean Euler residual and the true θ could be uncovered. Often, the mean is used to estimate the discount factor, β , (fixed at the true value here) and this estimate might just pick up this difference. In any case, since the bias is positive, and I am suggesting using the estimate as an upper bound, this has not been investigated further.

For completeness, the same exercise has been performed in a deterministic version of the model in which households have perfect foresight regarding arrival of children. Using $\Delta \bar{\mathbf{z}}_t$ as instrument still produce an upper bound, as shown in Figure A.6 in Appendix A. As also shown in the four-period model, since children only affect households in which they arrive in the deterministic model, using the actual change in number of children, $\Delta \mathbf{z}_t$, can identify the effect of children when the effect is sufficiently large.

4.2.3 Using These Bounds in Practice

Choosing the age at which to split the sample into young and older households is not obvious. One choice could be to chose the age at which the average number of children starts to decline since the behavior of households should differ when children arrives from when they move since credit constraints affect younger households most. Alternatively, the age at which average net wealth is significantly larger than average income could be chosen since around this point (on average) households are less affected by credit constraints. Optimally, if the data availability allows, recursive estimation results successively including younger and older households as done in Figure 4 and 5, respectively, would supply information about the effects of children.

The results rests on the existence of credit constraints. However, as previously argued, if instead of credit constraints, households face a small but positive probability of receiving some low income shock, all results concerning the log-linearized Euler equation still hold including the bounds, as shown in Figure A.4 in Appendix A. This result supports that, even if credit constraints are *not* present, the log-linearized Euler equation produce inconsistent estimates (due to omitted higher order terms in the residual, see [Carroll, 2001](#)) and can be utilized to estimate bounds on the effect of children on consumption.

One crucial assumption when calculating the bounds above is that of the researcher having knowledge on other structural parameters. Using the exact GMM estimation approach, both the discount factor, β , and the relative risk aversion, ρ , should be estimated simultaneously or

qualified guesses on these parameters should be used. Log-linearized Euler equation estimation requires information only on the risk aversion parameter. Therefore, the bounds vary with the choice of other structural parameters. This is a drawback but varying these parameters in “accepted” ranges would then produce a set of bounds with more or less information on the size of the effect of children.

Another assumption that has been invoked throughout is homoscedasticity of the Euler residuals. If, for example, different groups have different income variance or income growth rates the structural Euler residuals will be heteroscedastic. If ignored, this will invalidate the log-linearized Euler estimation approach since higher order moments will be present in the mean of the log-transformed Euler residual, $\tilde{\epsilon}$. [Browning, Ejrnæs and Alvarez \(2010\)](#) show evidence that income processes are characterized by significant heterogeneity indicating failure of the homoscedasticity assumption.

Measurement error in observed consumption measures have been ignored throughout. [Runkle \(1991\)](#) finds evidence that changes in log food consumption in the PSID is more than 70 percent measurement error. GMM estimators are notoriously vulnerable to measurement error and [Alan, Attanasio and Browning \(2009\)](#) propose two GMM estimators in which log-normal multiplicative measurement error in consumption is allowed for. Both estimators, however, rely on the availability of panel data and must assume log normal measurement error. [Alan and Browning \(2010\)](#) supply a simulation method (SRE) in which the Euler residual is fully parametrized along with the measurement error in consumption. Their method does not rely on panel data but cannot handle credit constraints and requires researchers to fully parametrize the the measurement error (and other unknown processes).²⁰

It is important to stress that throughout this paper, as in the rest of the literature, income is assumed *independent* of household composition. If income depend on household composition, the results will change. Although allowing income to vary with household composition is an interesting avenue for future research, I have not pursued that here.

An alternative route to estimating bounds is to utilize the moment inequality rather than the equality in the GMM estimator (2). Assuming that an instrument is potentially *positively* correlated with the Euler residual, $\mathbb{E}[(\epsilon - 1)'Z] \geq 0$ could be used as a moment inequality to estimate bounds ([Moon and Schorfheide, 2009](#)). This approach is very interesting for future research but I do not pursue that strategy here.²¹

I supply an alternative estimation strategy to estimate *all* structural parameters of the underlying model simultaneously. The estimation strategy does not rely on the availability of panel data and can accommodate a variety of measurement error specifications. The framework require information on the age of children, the level of resources inside the household (wealth and income) and consumption measures.

²⁰The measurement error parametrization proposed in [Alan and Browning \(2010\)](#) is extremely numerically unstable since three exponential functions are nested to insure positivity of several parameters.

²¹I am grateful to Dennis Kristensen for pointing this out to me.

5 Structural Estimation Strategy

In this section, I formulate a novel structural M-estimator to uncover parameters of intertemporal consumption choice models, analyzed throughout this study. I propose a continuous version of the Nested Fixed Point (NFXP) estimation approach, suggested by Rust (1987), that is consistent and asymptotically normally distributed under fairly standard regularity conditions. Specifically, for a given set of K structural parameters, Θ , the model is solved recursively for all combinations of household composition. This yields optimal consumption as a function of resources, permanent income and household composition, $\{C_t^*(M_t, P_t, \mathbf{z}_t|\Theta)\}_1^T$.²² In principle, Mathematical Programming with Equilibrium Constraints (MPEC), proposed by Su and Judd (2012) could be used to estimate parameters. However, as discussed in Jørgensen (2013), because the model in the present study is a finite horizon (life cycle) model with a large state space, MPEC most likely would be much slower than the NFXP.

This approach requires numerically solving for optimal policy rules from a full specified structural model. This is a drawback in the sense that *all* elements of the economic environment should be explicitly modeled. For example, the process of children arriving and moving over the life cycle is fully specified in the present model forcing a concrete choice of stochastic versus deterministic or endogenous arrival of children. I discussed this choice in Section 3. It can also be seen as an advantage because researchers can specify the environment exactly according to underlying assumptions. For example, including a credit constraint is rather simple in this estimation approach, while none of the estimators discussed so far can handle credit constraints.

GMM and Maximum Likelihood (ML) estimation are contained in the general framework and structural parameters can be estimated on *cross section data*. This makes the estimation approach extremely flexible and useful because structural parameters can be estimated using repeated cross sections which most surveys are.²³ The approach require, however, information on *i*) the age of children, *ii*) household income, *iii*) household wealth, and *iv*) consumption measures. These requirements may seem strong but increasingly many countries register these information on the individual level.

Let $\mathcal{O} = (M, P, C, \mathbf{z})^{data} \in \mathbb{O}$ denote *observed* information where $\mathbb{O} \subset \mathbb{R}^{\dim(\mathcal{O})}$ and \mathcal{O}_{it} refers to household i in period t and \mathbb{R} are the real numbers. Define a function of the observed data and model solution, for a given set of parameters, as

$$\xi_{it}(\Theta) \equiv \xi(\mathcal{O}_{it}, C_t^*|\Theta),$$

in which observed data is used to infer the model predictions for each household-time observation. Specifically, for a given observation, \mathcal{O}_{it} , the associated prediction from the structural model can be found by interpolating the relevant policy function, referred to as $\check{C}^*(\mathcal{O}_{it}|\Theta)$ in examples below. Let

$$g_i(\Theta, \phi) \equiv g(\xi_i(\Theta), \phi),$$

denote a real-valued function taking as argument stacked time observations, $\xi_i(\Theta)$, in which ϕ

²²Consult Appendix B for details on the solution method applied to solve the model described in Section 3.

²³The CEX is an extensively used repeated cross section survey on consumption behavior. Information on age of children are, to my knowledge, not available for the CEX households.

contain K_ϕ additional parameters. All parameters are in a compact space, $(\Theta, \phi) \in \mathbb{C} \subset \mathbb{R}^{K+K_\phi}$, and $g : \mathbb{O} \times \mathbb{C} \rightarrow \mathbb{R}$ is, for all $\mathcal{O}_{it} \in \mathbb{O}$, continuous in (Θ, ϕ) .²⁴

The proposed estimator solves the problem

$$\min_{(\Theta, \phi) \in \mathbb{C}} N^{-1} \sum_{i=1}^N g_i(\Theta, \phi), \quad (8)$$

and, assuming an *unique* solution exists, is consistent by the uniform weak law of large numbers. Assume that *i*) the true parameters, (Θ_0, ϕ_0) , are in the interior of \mathbb{C} , *ii*) the gradient, $\mathbf{s}(\Theta, \phi)$, and hessian, $\mathbf{H}(\Theta, \phi)$, of g exists in this interior point, *iii*) the gradient has mean zero and finite second order moment, *iv*) the mean hessian is positive definite, and *v*) each element of the hessian is bounded. Then, the estimator is asymptotically normally distributed around the true parameter with asymptotic variance of $\mathbf{A}^{-1} \mathbf{B} \mathbf{A}^{-1} / N$, where $\mathbf{A} \equiv \mathbb{E}[\mathbf{H}(\Theta, \phi)]$ and $\mathbf{B} \equiv \mathbb{E}[\mathbf{s}(\Theta, \phi) \mathbf{s}(\Theta, \phi)']$ (Wooldridge, 2002).

Optimal behavior is in general found numerically and the objective function in (8) is an *approximation* to the exact objective function. Fernández-Villaverde, Rubio-Ramírez and Santos (2006) show, in a likelihood framework, that as long as the numerical approximation converges to the *unique* exact solution, the approximated likelihood function converges uniformly to the exact likelihood function. This provides the strong result that parameters estimated by (8) are consistent and asymptotically normally distributed even when the solution, C^* , is found numerically.²⁵

To illustrate the flexibility of the estimator, I present concrete examples in which assumptions often invoked in the literature are implemented in the framework above. Example 1 illustrates how the estimator can estimate structural parameters if consumption is contaminated with additive normally distributed measurement error. Readers who feel uncomfortable with the normality assumption in Example 1 could think of the estimation problem as one of non-linear least squares. Alternative distributional assumptions could be implemented or the absolute difference could be minimized ($g_i(\Theta) = \sum_t^{T_i} |\xi_{it}(\Theta)|$), yielding an estimator more robust to outliers.

Example 1 (Additive Normal Measurement Error). If consumption data is contaminated with *iid* additive $\mathcal{N}(0, \sigma_\xi^2)$ measurement error, then letting

$$\begin{aligned} \xi_{it}(\Theta) &= C_{it}^{data} - \check{C}^*(\mathcal{O}_{it}|\Theta), \\ g_i(\Theta, \sigma_\xi) &= T_i \log(2\pi\sigma_\xi^2) + \frac{1}{2\sigma_\xi^2} \sum_t^{T_i} \xi_{it}(\Theta)^2, \end{aligned}$$

produce structural parameters that maximize the likelihood of observed data being generated from the structural model.

Jørgensen (2013) shows that the estimation approach outlined in Example 1 can uncover

²⁴Also, for each $(\Theta, \phi) \in \mathbb{C}$, g should be Borel measurable on \mathbb{O} .

²⁵Ackerberg, Geweke and Hahn (2009) correct a result (Proposition 2) of Fernández-Villaverde, Rubio-Ramírez and Santos (2006) stating that for the *approximated* likelihood to converge to the exact one the approximation error should decrease faster than the increase in observations. Ackerberg, Geweke and Hahn (2009) reassuringly show that this is *not* the case.

parameters like the relative risk aversion, ρ , from similar models. For completeness, Table A2 reports mean (and standard deviation) of θ estimates from 50 independent simulations in which measurement error is added with a known variance of one. The estimation approach uncovers the true parameter, θ_0 , in even small samples.

Consumption is often assumed to be observed with *multiplicative* measurement error (Gourinchas and Parker, 2002; Alan, Attanasio and Browning, 2009 are examples). The proposed framework can easily encompass this situation by letting $\xi_{it}(\Theta) = \log C_{it}^{data} - \log \check{C}^*(\mathcal{O}_{it}|\Theta)$ and letting $g(\cdot)$ correspond to a distributional assumption. If panel data is available, the measurement error can be allowed to vary systematically across households, as illustrated in Example 2 in which the multiplicative measurement error are heterogeneous and arbitrarily distributed with a constant variance, σ_ξ .

Example 2 (Multiplicative Heterogeneous Measurement Error). If consumption data is contaminated with multiplicative measurement error systematically different across households, then

$$\begin{aligned}\xi_{it}(\Theta) &= \log C_{it}^{data} - \log \check{C}^*(\mathcal{O}_{it}|\Theta), \\ g_i(\Theta, \sigma_\xi) &= \sum_t^{T_i} |\Delta \xi_{it}(\Theta)|,\end{aligned}$$

produce consistent estimates of Θ and σ_ξ^2 , independent of the distribution of the measurement error. If, as assumed in Alan, Attanasio and Browning (2009), the measurement error is log-normally distributed, letting $g_i(\Theta, \sigma_\xi) = T_i \log(4\pi\sigma_\xi^2) + \frac{1}{4\sigma_\xi^2} \sum_t^{T_i} (\Delta \xi_{it}(\Theta))^2$ would produce a consistent ML estimator in this case.

Most of the examples are ML estimators but GMM estimators can easily be constructed within this framework. Define p moments or auxiliary parameters (ap) from the data, λ , and corresponding moments from the predictions of the model, $\lambda(\Theta)$, and let $\xi_{it}(\Theta) = \lambda - \lambda(\Theta)$ be the difference between these moments. Formulating $g_i(\Theta, \phi) = \xi(\Theta)'W(\phi)^{-1}\xi(\Theta)$ as the quadratic sum of differences, where $W(\phi)$ is some weight matrix, produce a GMM estimator within the framework of equation (8).

The general framework also offers a straight forward variance adjustment of two-step estimators. Often some parameters of the model is calibrated or estimated in a first-step procedure and taken as given in the subsequent estimation of parameters of interest. Wooldridge (2002, p. 361) discuss variance-adjustment and show applicable formulas for the general estimator proposed here.

6 Empirical Applications: Danish Register Data and the PSID

I implement the suggested bounds and structural estimation strategy using two different data sources. First, high quality Danish administrative register data for the entire Danish population is used providing reliable longitudinal information on relevant variables. Danish data has not previously been used to uncover the effect of children on household consumption so to present results comparable to an existing literature, I also present results using the extensively used PSID panel survey of US households.

6.1 The Danish Registers

The high quality Danish administrative registers utilized here cover the entire population from the period 1987-1996. All information are based on third party reports with little additional self-reporting. All self-reporting are subject to possible auditing giving reliable longitudinal information on household characteristics, wealth and income.

Household consumption is not observed in the registers and is imputed using a simple budget approach,

$$C_t = Y_t - \Delta A_t, \quad (9)$$

where Y_t is disposable income, A_t is end-of-period net wealth, and thus ΔA_t proxies savings. This imputation method is evaluated on Danish data in [Browning and Leth-Petersen \(2003\)](#) and found to produce a reasonable approximation. The resulting consumption measure will, however, include some durables such as dishwasher etc. that might not be included in survey measures of consumption and should be kept in mind when comparing results.

Net wealth consists of stocks, bonds, bank deposits, cars, boats, house value for home owners and mortgage deeds net of total liabilities. The house value is assessed by the tax authorities for tax purposes. The amounts held in specific stocks are not known, only the total value of all stocks is.

Pension wealth is not included in the wealth measure. Information on pension accounts are not available for most of the cohorts studied here and the resulting net wealth is, therefore, slightly underestimated. [Jørgensen \(2007\)](#) shows that pension fund deposits are minor for young households and increase until the age of retirement. Further, pension funds are non-liquid until retirement and only few withdraw pension funds prematurely since heavy taxation leaves only 40% of prematurely withdrawn funds available for consumption purposes. Prematurely withdrawn pension funds are included in the disposable income and since I focus on pre-retirement behavior exclusion of pension wealth is expected to have only minor effects on the results.

Disposable income include all labor market and non-labor market income net of all taxes. Transfers, such as child care subsidies and unemployment benefits, are also included such that disposable income measures the flow of resources available for consumption.

I restrict attention to continuously married and cohabiting couples in which the age of the wife aged 25 to 59 to mitigate selection issues regarding educational and retirement choices. To increase homogeneity of households, I restrict the spousal age difference to be no more than four years. Households in which one adult is self-employed or out of the labor market are dropped from the analysis. Hence, households with one or more unemployed individuals are included but retirees are not. This is in order to construct a sample that is comparable to the existing literature. Extreme or missing observations are also excluded from the analysis leaving an unbalanced panel of 201,618 households observed in at most 9 periods with a total of 1,281,952 observations. Table A1 in Appendix A presents the sample selection criteria and their impact on the population.

6.2 The PSID

The Panel Study of Income Dynamics (PSID) contains information on food consumption in and out of home and has been used intensively for several purposes, including estimation of the effect of children on consumption. To study the evolution and link between income and consumption inequality over the 1980s, [Blundell, Pistaferri and Preston \(2008\)](#) impute total non-durable consumption for PSID households using food consumption measures in the CEX. I use their data set and measure of total consumption and refer the reader to their discussion of the PSID data.

The sample period used in [Blundell, Pistaferri and Preston \(2008\)](#) is 1978 to 1992 and only male headed continuously married couples are used. The years 1987 and 1988 is not used since consumption measures were not collected these years. Since the present study focus on the effect of children on household consumption, and I want the sample to be comparable to the Danish sample, I restrict the sample to cover 25 to 59 year old households and link this to the age of the wife.²⁶ Further, I drop households in which the age difference between husband and wife is larger than four years and the number of children is greater than three. I also drop households who have children before age 15 or later than 40. This is a very small fraction and an unbalanced panel of 2,350 households observed for at most 13 periods are in the final sample yielding a total of 17,005 non-missing observations.

Household resources are composed of after tax household income of both spouses and household assets. The PSID does not contain information on liabilities and, therefore, I do not include house value in the measure of assets. The measure of resources in the PSID, therefore, differs significantly from measure constructed for the Danish data but should still provide a reasonably good proxy for consumption possibilities within a household.

The structural estimation approach outlined above requires knowledge on the age of each of the (potential) three children. This information is not in the original [Blundell, Pistaferri and Preston \(2008\)](#) data but is linked through the age of birth of each child in the PSID survey.

6.3 Calibrations

Some parameters of the model are calibrated and Table 4 reports the values and sources for these parameters. The real gross interest rate, R , is calibrated to 1.03 for both countries, following most of the literature, and the income growth rate, G_t , is estimated by average changes in log income, for different age groups. Figures A.9 and A.10 in Appendix A reports the income growth rate profile for Danish and US consumers, respectively.

When retired, US households are assumed to experience a constant decrease in permanent income of 5 percent while income of Danish retirees are constant. This does not affect the results significantly since the value of retirement, γ , will adjust accordingly. The exogenous drop in permanent income when households retire, \varkappa , is calibrated to 90 percent in Denmark based on the median couple in the study by [Ministry of Finance \(2003\)](#). This implies a rather high level of income from transfers post retirement and stems from generous public transfers and private pension funds. Since the pension system is less generous in the US, I calibrate this

²⁶[Blundell, Pistaferri and Preston \(2008\)](#) use households in which the husband is aged 30 to 65.

drop to be larger, 20 percent, when calibrating the model to the PSID data.

Table 4 – Calibrated Parameters.

	Denmark		PSID	
	Value	Source	Value	Source
G_t	Fig. A.9	Own calculations: see text	Fig. A.10	Own calculations: see text
R	1.03	Gourinchas and Parker (2002)	1.03	Gourinchas and Parker (2002)
κ	-0.6	Own calculations: see text	0.00	Self imposed constraint
\wp	0.10	Own calculations: see text	0.003	Gourinchas and Parker (2002)
μ	0.30	Own calculations: see text	0.00	Gourinchas and Parker (2002)
\varkappa	0.90	Ministry of Finance (2003)	0.80	Own calculations: see text
G_{ret}	1.00	Own calculations: see text	0.95	Own calculations: see text

The low transitory income shock is calibrated such that with 0.3 percent probability US households receive zero income ($\mu_{US} = 0$ and $\wp_{US} = .003$), following [Gourinchas and Parker \(2002\)](#). This implies that households would never want to leave zero resources to the next period in fear of having to consume zero with a dis-utility of negative infinity (hence κ does not affect the behavior and is set to zero). The social security system in Denmark is more compatible with a 10 percent risk of income being reduced to 30 percent. Danish households are allowed to be net-borrowers by 60 percent of annual permanent income. These three values are somewhat arbitrary and have been chosen to provide reasonable fit in the bottom distribution of resources for households below age 40 and do not influence the results significantly.

The permanent and transitory income shock variances are estimated following the approach in [Meghir and Pistaferri \(2004\)](#). First, I run a regression of income on year dummies and the resulting log residual income, \tilde{y}_t , is used to calculate the permanent and transitory income shock variances as

$$\begin{aligned}\hat{\sigma}_\eta^2 &= \text{cov}(\Delta\tilde{y}_t, \Delta\tilde{y}_{t+1} + \Delta\tilde{y}_t + \Delta\tilde{y}_{t-1}), \\ \hat{\sigma}_\varepsilon^2 &= -\text{cov}(\Delta\tilde{y}_t, \Delta\tilde{y}_{t+1}).\end{aligned}$$

Table 5 presents the estimated variance components for both data sources. The permanent income shocks are found to be more volatile for high skilled households, a robust result in the literature. The variance of transitory income shocks is, however, often found to be lower for high skilled households. I find the opposite here. The permanent income shocks account for slightly more of the variation in income relative to the transitory shocks ($\hat{\sigma}_\eta > \hat{\sigma}_\varepsilon$) in the PSID while most existing studies report the opposite result.²⁷ This is most likely due to the fact that i only remove year effects while most other studies include other “deterministic” components such as the number of children and household age ([Carroll and Samwick, 1997](#)). However, since I want to keep all of these aspects in the income and consumption measure, I believe it will be more comparable to also include variation from such factors in the variance measure. As a robustness check, the estimated income shock variances from [Gourinchas and Parker \(2002\)](#) has been used

²⁷[Blundell, Pistaferri and Preston \(2008\)](#) report $\sigma_\eta^2 \in [.0057, 0.0333]$ and $\sigma_\varepsilon^2 \in [.0190, 0.0753]$ depending on the combination of year, cohort and educational background, using the PSID. [Gourinchas and Parker \(2002\)](#), also using the PSID, calibrate $\sigma_\eta^2 = 0.0212$, $\sigma_\varepsilon^2 = 0.0440$.

without any significant changes to the results (see Table A5).

Table 5 – Permanent and Transitory Income Shock Variances.						
	All		Low skilled		High skilled	
	Est	SE	Est	SE	Est	SE
<i>Danish Registers</i>						
$\hat{\sigma}_{\eta}^2$	0.0054	(0.000096)	0.0049	(0.000113)	0.0062	(0.000173)
$\hat{\sigma}_{\varepsilon}^2$	0.0072	(0.000156)	0.0059	(0.000167)	0.0095	(0.000315)
<i>PSID</i>						
$\hat{\sigma}_{\eta}^2$	0.0785	(0.003898)	0.0756	(0.005973)	0.0815	(0.005004)
$\hat{\sigma}_{\varepsilon}^2$	0.0510	(0.004452)	0.0476	(0.005374)	0.0543	(0.007086)

Notes: Estimates are based on the approach in [Meghir and Pistaferri \(2004\)](#). Robust standard errors in parenthesis.

The Danish income variances are an order of magnitude smaller than those for the US. This is most likely due to the generous social welfare system and progressive taxation in Denmark. Denmark has a relatively high minimum wage of around \$20 per hour (in 2010) reducing the volatility in permanent and transitory income shocks compared to, e.g., the US and the Danish tax system is one of the worlds most progressive tax schedules with a marginal tax rate of more than 60 percent in 2010 for top earners. Around 40 percent where top earners in 2010, reducing the dispersion in disposable income significantly compared to, e.g., the US.²⁸

Unobserved permanent income, P_t , is uncovered by the Kalman Filter applied to each household's income process. Consult Appendix D for a description of the implementation. The arrival rate of infants are estimated as a simple logit model on age dummies for each educational group and number of children already present in the household. The arrival probabilities using Danish registers and the PSID are presented in Figures A.7 and A.8, respectively.

6.4 Structural Estimation Results

For both data sources, I use the raw series only corrected for year-dummies through regressions to avoid removing valuable variation over the life cycle that might be correlated with children. The baseline assumption is that consumption, normalized with permanent income, is observed with additive normal distributed measurement error. Results are robust to the distributional assumption as robustness checks suggest.

Several versions of the model are estimated using each data set. First a model without any household compositional effects, then a functional form of the taste shifter similar to existing literature, $v(\mathbf{z}_t, \theta) = \exp(\theta \# \text{children})$ is estimated, and, finally, a flexible functional form is

²⁸The Danish administrative registers also tend to be less noisy compared to surveys ([Browning and Leth-Petersen, 2003](#)), reducing the transitory income shock variance.

implemented,

$$\begin{aligned}
v(\mathbf{z}_t, \theta) = & 1 + \theta_{11}\mathbf{1}_{\{\text{Age of child 1} \in [0,10]\}} + \theta_{12}\mathbf{1}_{\{\text{Age of child 1} \in [11,21]\}} \\
& + \theta_{21}\mathbf{1}_{\{\text{Age of child 2} \in [0,10]\}} + \theta_{22}\mathbf{1}_{\{\text{Age of child 1 and 2} \in [11,21]\}} \\
& + \theta_{31}\mathbf{1}_{\{\text{Age of child 3} \in [0,10]\}} + \theta_{32}\mathbf{1}_{\{\text{Age of child 1, 2 and 3} \in [11,21]\}},
\end{aligned}$$

allowing for an arbitrary children, age and scale effect.

Table 6 presents the estimation results for low and high skilled Danish and US households. The results are very similar across the two data sources, the estimated relative risk aversion and discount factor are in the range normally found in the literature, and the model fits the actual consumption age profiles quite well, as reported in Figures A.11 and A.12 in Appendix A. The elasticity of intertemporal substitution ($\text{EIS} = \rho^{-1}$), increase when children have a positive effect on marginal utility of consumption. This stems from the fact that when marginal utility of consumption increase in the presence of children, households are willing to substitute more consumption to future periods when children arrives. This intertemporal substitution would be captured in ρ when θ is fixed at zero. The discount factor does not respond significant to inclusion of children.

As most of the literature, I find that high skilled are more risk averse and more patient than low skilled households using the Danish data. The opposite is the case for the PSID but stems most likely from the fact that several post retirement parameters (γ , κ , G_{ret}) are calibrated at the same value across educational groups for the PSID households. Also, the estimated value of ρ is affected by the calibrated income shock variances, as discussed above when parameters are calibrated. Since the focus here is on the estimation of θ , I have not investigated this further.

Turning to the parameters of interest, θ , both Danish and US households does not seem to be significantly affected by the presence of children. Although formal Likelihood Ratio (LR) tests reject that $\theta = 0$ for Danish households, the effect is economically very small and even negative for low skilled households.

For US households, the estimated effects are larger around 0.2 – 0.3, close to the estimate reported in [Attanasio, Banks, Meghir and Weber \(1999\)](#) and some of the estimates reported in [Alan, Attanasio and Browning \(2009\)](#). As in the latter study, the present estimates are very imprecise and the LR test also suggests that parameters relating to children could be ignored. Further, the fit of the estimated model *without* children is significantly better than the fit of the model with children, as shown in Figure A.12. In general, the estimates from the PSID is much less robust to starting values and the amount of measurement error also is much larger compared to the Danish register data.

There is a tendency to older children affecting consumption more than younger children, as found in [Browning and Ejrnæs \(2009\)](#) but, surprisingly and in contrast to their results, the estimated parameters suggest that there is *no* economies of scale. For example, for high skilled US households, the effect of having a second child older than 10 years when a child in this age group is already present increase the marginal value of consumption with more than the first child did.

Table 6 – Estimated Preference Parameters, Danish Registers and PSID.

		Danish Registers						PSID					
		Low skilled			High skilled			Low skilled			High skilled		
		(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
ρ	Risk aversion	2.316 (0.041)	2.363 (0.036)	2.385 (0.043)	2.639 (0.057)	2.626 (0.062)	2.634 (0.063)	2.051 (0.738)	1.747 (0.439)	1.596 NA	1.828 (0.470)	1.984 (1.061)	1.495 (0.020)
β	Discount factor	0.965 (0.000)	0.964 (0.000)	0.964 (0.000)	0.973 (0.001)	0.973 (0.001)	0.972 (0.001)	0.948 (0.021)	0.965 (0.009)	0.966 NA	0.931 (0.020)	0.939 (0.037)	0.948 (0.013)
γ	Retirement	1.454 (0.018)	1.492 (0.018)	1.491 (0.020)	1.251 (0.022)	1.245 (0.023)	1.265 (0.025)	1.300	1.300	1.300	1.300	1.300	1.300
σ_ξ	Meas. err.	0.468 (0.000)	0.468 (0.000)	0.468 (0.000)	0.490 (0.001)	0.490 (0.001)	0.490 (0.001)	1.343 (0.011)	1.342 (0.011)	1.342 (0.011)	3.582 (0.029)	3.582 (0.029)	3.582 (0.029)
$v(\mathbf{z}; \theta) = \exp(\theta' \mathbf{z})$													
θ	# of children		-0.017 (0.002)			0.004 (0.003)			0.224 (0.098)			0.326 (0.361)	
$v(\mathbf{z}; \theta) = 1 + \theta' \mathbf{z}$													
θ_{11}	1. child ≤ 10			-0.004 (0.007)			-0.008 (0.010)			0.117 (0.288)			-0.087 (0.549)
θ_{12}	1. child > 10			-0.031 (0.004)			0.002 (0.008)			0.353 (0.292)			0.247 (0.530)
θ_{21}	2. child ≤ 10			-0.034 (0.005)			-0.015 (0.008)			0.158 (0.221)			0.158 (0.521)
θ_{22}	2. child > 10			-0.006 (0.005)			0.000 (0.008)			0.023 (0.313)			0.320 (0.653)
θ_{31}	3. child ≤ 10			-0.005 (0.009)			0.022 (0.012)			0.535 NA			-0.016 (0.676)
θ_{32}	3. child > 10			0.019 (0.013)			0.021 (0.017)			0.277 (0.483)			0.630 (0.369)
$-\mathcal{L}(\Theta)$		0.46536	0.46533	0.46529	0.49868	0.49863	0.49862	0.83299	0.83269	0.83263	1.13714	1.13710	1.13706
LR [p -val]			67.1[0.00]	153.4[0.00]		57.1[0.00]	68.8[0.00]		8.7[0.00]	10.4[0.11]		1.6[0.21]	2.9[0.82]
Observations			851, 249			430, 703			8, 333			8, 672	

Notes: Standard errors are based on the inverse of the hessian. "NA" refers to Not Available and stems from the hessian being close to singular and the resulting parameter variances are negative. This indicates that the objective function is flat within numerical precision and the estimates and standard errors should be interpreted with caution. For US households, using the PSID, the retirement motive is fixed across educational groups to be 1.3 based on results for Danish households.

The results are very robust. Changing the objective function to a Logistic distribution or absolute differences does not affect the estimated effect of children significantly, as reported in Table A3. Further, allowing for *heterogeneous multiplicative distribution-free measurement error* (Example 2 in Section 5) does not change the results (see Table A5). Also, calibrating the transitory and permanent income variance to those used in [Gourinchas and Parker \(2002\)](#) does not change the results (Table A5).

The difference between the results across US and Denmark could be due to differences in the consumption measures used. The Danish consumption measure effectively includes some durables such as dishwashers and other smaller household appliances while the US measure is imputed from food consumption measures in the PSID and CEX. This difference is likely to produce a measure including more than non-durable consumption for Denmark and a measure of consumption in the PSID to closely related to food consumption, a consumption component likely more affected by the presence of children than total non-durable consumption. Also, the labor market responses to arrival of children is likely to differ across the two countries.

The assumption that children arrive probabilistically is expected to produce lower estimates of the effect of children, compared to deterministic or endogenous arrival of children. This is because *all* households within a given age group have the same expected children-related expenditures and, therefore, suggest that *all* households should decrease consumption in anticipation of children. If this is not how households perceive the world, the estimate of θ would be forced downwards by households who *know* that they will have children, say, late in the life cycle and therefore does not increase savings as much as the model would suggest when young.

Table A4 reports unchanged estimation results from a deterministic version of the model, in which households know with perfect foresight how many and when children will arrive, using the PSID. Since this model requires knowledge on *completed* fertility, I made the crude assumption that observed fertility is completed fertility.²⁹ Since endogenous fertility choice is in between the two opposite extreme models (stochastic versus deterministic arrival of children) producing identical results, the results is expected not to change if children where endogenously chosen.

The effect of children on consumption is surprisingly low. Inspecting the raw consumption data show an intriguing pattern that might explain this result: Total non-durable consumption is not affected much by children, but the *composition* of consumption goods might be. Figure A.13 in Appendix A reports consumption age profiles for PSID households who have at least one child at age 30 and households who are childless at age 30. Profiles are shown for (a) food at home, (b) food out, (c) total food consumption, and (d) the imputed non-durable consumption constructed in [Blundell, Pistaferri and Preston \(2008\)](#) and used throughout the present study.

Expenditures on food at home is on average higher for households who have children while food out is on average higher for households who have no children. Total food consumption and non-durable consumption does, however, not differ significantly across households who have children and childless households at age 30. This indicates that children might shift consumption from luxury goods (food out) towards necessary goods (food home) while leaving total expenditures almost unaffected.³⁰

²⁹ Alternatively, outside the scope of this paper, an estimated completed fertility could be used (see [Browning and Ejrnæs, 2009](#)).

³⁰ [Aguiar and Hurst \(2013\)](#) argue that food out is related to labor market activities and the drop in food out

6.5 Reduced Form Bounds

Table 7 – Log-Linear Euler Equation Estimation Results, $\widehat{\rho^{-1}\theta}$.

	Low Skilled				High Skilled			
	Lower [†]	Upper [‡]	$\Delta\mathbf{z}_t$	$\Delta\bar{\mathbf{z}}_t$	Lower [†]	Upper [‡]	$\Delta\mathbf{z}_t$	$\Delta\bar{\mathbf{z}}_t$
Denmark	0.027 (0.005)	0.145 (0.011)	0.031 (0.003)	0.210 (0.009)	0.020 (0.006)	0.183 (0.015)	0.043 (0.004)	0.210 (0.012)
PSID	0.019 (0.037)	0.105 (0.073)	0.079 (0.024)	0.158 (0.060)	0.058 (0.024)	0.153 (0.186)	0.076 (0.027)	0.205 (0.143)

Notes: Reported estimates are $\widehat{\rho^{-1}\theta}$ with robust standard errors in parenthesis. Estimates from PSID is based on food consumption at home. Estimates using Danish register data is based on imputed consumption using income and wealth information.

[†] Lower bound is the estimate from a log-linearized Euler equation using only households in which the wife is age 45 or above.

[‡] Upper bound is the estimate from a log-linearized Euler equation using $\Delta\bar{\mathbf{z}}_t$ as instrument and restricting the sample to households in which the wife is age 45 or below.

Table 7 reports the estimated bounds of $\rho^{-1}\theta$ for the Danish and US data using the log-linearized Euler equation (3). Recall that Section 4.2 argued that a lower bound on θ can be estimated using the household level change in number of children ($\Delta\mathbf{z}_t$) while restricting the sample to include only older households. The estimated lower bounds are reported in column one and five for low and high skilled households, respectively. An upper bound can be found by using the cohort-average number of children ($\Delta\bar{\mathbf{z}}_t$) as an instrument while restricting the sample to younger households. The resulting upper bounds are reported in columns two and six.

As reference, and to confirm that the PSID sample used herein and Danish data are very similar to the data used in the literature, I also report estimates including all households. The estimates are very similar to estimates reported in the literature. [Alan, Attanasio and Browning \(2009\)](#) using the household level change in number of children, $\Delta\mathbf{z}_t$, report an estimate of .045 and I estimate .079, both using the PSID. The estimate for Denmark is slightly lower and even closer at around .03 – .04. When using the cohort-average number of children as an instrument, the estimates increase significantly to around .2, very similar to estimates reported in the literature ([Attanasio, Banks, Meghir and Weber, 1999](#) estimates $\rho^{-1}\theta$ to be .21, using the CEX).

Assuming that the relative risk aversion is as estimated in column (2) Table 6, bounds of the effect of children can be found as $\hat{\rho} \times \widehat{\rho^{-1}\theta}$ as reported in Table 8.³¹ Although the bounds are rather wide, they suggest that what have typically been reported in the literature, using Euler equation estimation methods, is close to (and outside) the upper bound, while the structural estimation results I present is closer (and below) to the lower bounds. The effect of children, thus, might be much smaller than what have previously been assumed in the literature. Specifically, the reported estimate of $\rho^{-1}\theta$ in [Attanasio, Banks, Meghir and Weber \(1999\)](#) of 0.21 and ρ^{-1} of 0.64 produce an effect of children on consumption around $\hat{\theta} \approx 0.33$, close to the upper bound

and increase in food home over the life cycle is interpreted as being caused by lower labor market participation of older households. The suggestive evidence I report offer an alternative explanation and interesting avenues for future research.

³¹I use the estimated risk aversion parameters from column (2) because the bounds are derived from a model with that taste shifter specification, $v(\mathbf{z};\theta) = \exp(\theta'\mathbf{z})$, typically applied in the literature.

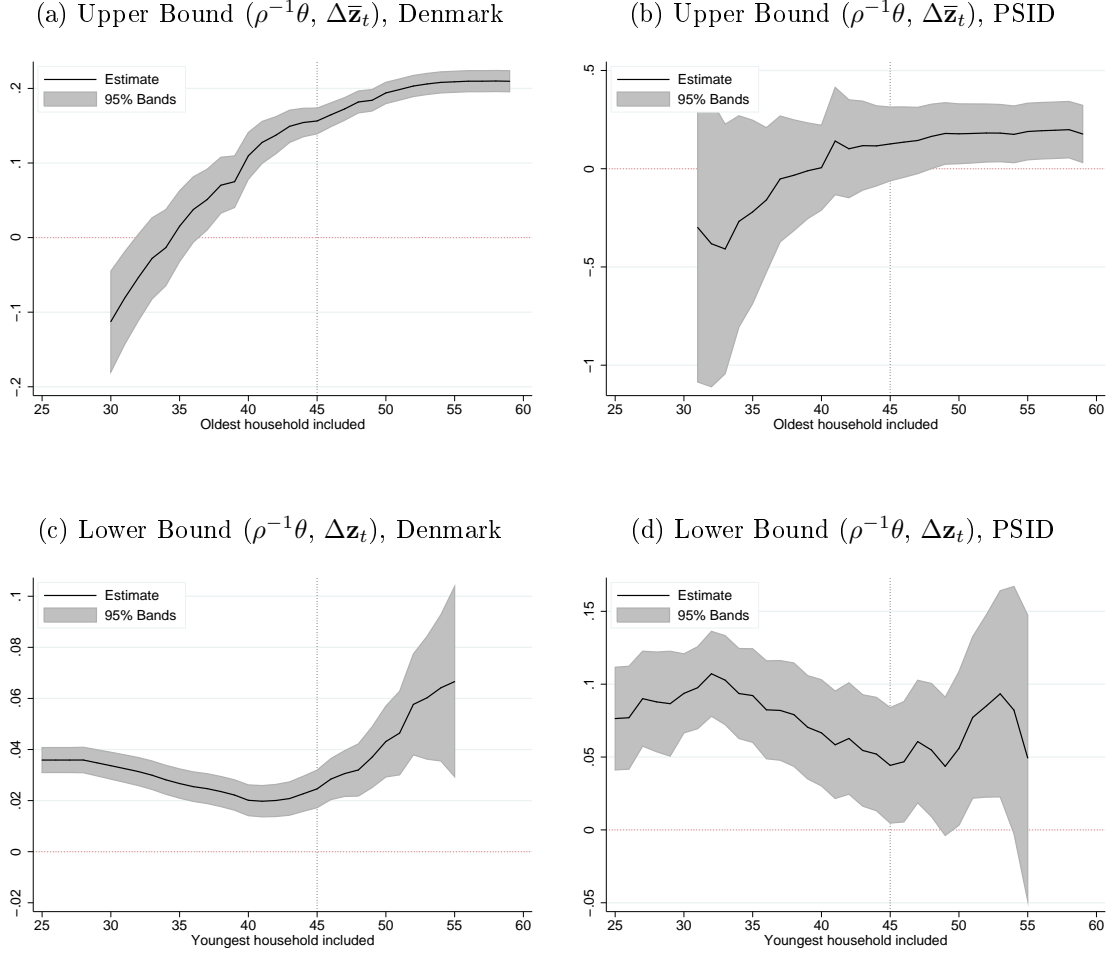


Figure 6 – Estimated Upper and Lower Bounds for Varying Cut-off Ages.

of high skilled households using the PSID.

The bounds reported in Table 7 and 8 have been estimated using 45 as the age at which the sample is split between young and older households. As discussed above, this is subject to a fair amount of arbitrariness. Figure 6 presents the lower and upper bounds of $\widehat{\rho^{-1}\theta}$ for varying cut-off ages. The top panel, (a) and (b), are constructed such that increasingly older households are included from age 25 through 59 while in the bottom panel, (c) and (d), increasingly younger households are included from age 59 through 25.

The lower bounds form an U-shape as increasingly younger households are included similar to the ones found on simulated data (see Figure 4). In contrast, the upper bounds differ from the ones found using simulated data (Figure 5). Including increasingly older households seem to monotonically increase the estimated effect of children. This could, e.g., arise from older children having a larger impact on marginal utility or younger cohorts being less affected by the presence of children relative to older cohorts.

The bounds are derived from a stylized model and other significant life cycle motives such as labor market participation and consumption of durables such as housing is likely to differ over the life cycle. Also, if children affect household income, the bounds will be invalid. Investigating

Table 8 – Bounds of the Effect of Children on Consumption, $\theta \in [\underline{\theta}, \bar{\theta}]$.

	Danish Registers		PSID	
	Low skilled	High skilled	Low skilled	High skilled
Lower bound, $\underline{\theta}$	0.064	0.053	0.033	0.115
Upper bound, $\bar{\theta}$	0.343	0.481	0.183	0.304

Notes: Reported estimates are $\hat{\rho} \times \widehat{\rho^{-1}\theta}$ where $\hat{\rho}$ is the estimated risk aversion parameter from columns (2) in Table 6 and $\widehat{\rho^{-1}\theta}$ is the estimated lower and upper bounds from Table 7.

these potentially important factors and choices in the present model is fascinating for future research, but is outside the scope of this paper.

7 Concluding Discussion

Throughout this study, the effect of children on non-durable consumption has been analyzed. Building on a standard buffer stock model in which the marginal utility is allowed to depend on the number and age of children, I show that reduced form estimators based on the consumption Euler equation produce inconsistent estimates of the effect of children if households face credit constraints. Many of the results hold even if households do not face credit constraints but instead face a positive probability of receiving a low income shock, a specification used in, e.g., [Carroll \(1997\)](#) and [Gourinchas and Parker \(2002\)](#).

Utilizing the Euler equation bias, I provide a tractable method to estimating a lower and upper bound on the effect of children on household consumption when credit constraints affect household behavior. I show that using only young households and a suitable instrument, an upper bound of the effect of children on consumption can be estimated and if the effect of children is large enough, this upper bound (almost) equals the true parameter. In the same line of thought, I argue that using only older households can be used to estimate a lower bound.

Finally, I have proposed a flexible structural estimation strategy in which the economic environment of consumers is fully specified and optimal consumption behavior is found numerically for given parameter values. This approach overcomes limitations of the bound estimation approach such as simultaneous estimation of *all* relevant preference parameters, measurement error in consumption and potentially binding credit constraints. Using this approach, I estimate the effect of children on household consumption using both the PSID for the US and high quality Danish administrative register data of the entire population. Results suggest that the effect of children reported in the existing literature is in the range of the upper bound while the estimates from the structural estimation approach are close to the lower bound.

Several interesting avenues for future research remains. The finding of small effects of children on non-durable consumption is likely to camouflage significant shifts in the composition of expenditures within a household. Specifically, the arrival of children may shift expenditures from luxury goods towards more necessary goods while leaving the total expenditures almost unaffected.

References

- ACKERBERG, D., J. GEWEKE AND J. HAHN (2009): "Comments on "Convergence Properties of the Likelihood of Computed Dynamic Models",” *Econometrica*, 77(6), 2009–2017.
- ADDA, J. AND R. COOPER (2003): *Dynamic Economics: Quantitative Methods and Applications*. MIT Press.
- AGUIAR, M. AND E. HURST (2013): “Deconstructing Life Cycle Expenditure,” *Journal of Political Economy*, 121(3), 437–492.
- ALAN, S., O. ATTANASIO AND M. BROWNING (2009): “Estimating Euler equations with noisy data: two exact GMM estimators,” *Journal of Applied Econometrics*, 24(2), 309–324.
- ALAN, S. AND M. BROWNING (2010): “Estimating Intertemporal Allocation Parameters using Synthetic Residual Estimation,” *The Review of Economic Studies*, 77(4), 1231–1261.
- ATTANASIO, O. P., J. BANKS, C. MEGHIR AND G. WEBER (1999): “Humps and Bumps in Lifetime Consumption,” *Journal of Business & Economic Statistics*, 17(1), 22–35.
- ATTANASIO, O. P. AND M. BROWNING (1995): “Consumption over the Life Cycle and over the Business Cycle,” *The American Economic Review*, 85(5), 1118–1137.
- ATTANASIO, O. P. AND H. LOW (2004): “Estimating Euler equations,” *Review of Economic Dynamics*, 7(2), 406 – 435.
- ATTANASIO, O. P. AND G. WEBER (1995): “Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey,” *The Journal of Political Economy*, 103(6), 1121–1157.
- BICK, A. AND S. CHOI (2013): “Revisiting the effect of household size on consumption over the life-cycle,” *Journal of Economic Dynamics and Control*, 37(12), 2998–3011.
- BLAU, D. M. (2008): “Retirement and Consumption in a Life Cycle Model,” *Journal of Labor Economics*, 26(1), 35–71.
- BLUNDELL, R., M. BROWNING AND C. MEGHIR (1994): “Consumer Demand and the Life-Cycle Allocation of Household Expenditures,” *The Review of Economic Studies*, 61(1), 57–80.
- BLUNDELL, R., L. PISTAFERRI AND I. PRESTON (2008): “Consumption inequality and partial insurance,” *American Economic Review*, pp. 1887–1921.
- BROWNING, M., A. DEATON AND M. IRISH (1985): “A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle,” *Econometrica*, 53(3), pp. 503–544.
- BROWNING, M. AND M. EJRNÆS (2009): “Consumption and Children,” *The Review of Economics and Statistics*, 91(1), 93–111.
- BROWNING, M., M. EJRNÆS AND J. ALVAREZ (2010): “Modelling Income Processes with Lots of Heterogeneity,” *Review of Economic Studies*, 77(4), 1353–1381.

- BROWNING, M. AND S. LETH-PETERSEN (2003): “Imputing Consumption from Income and Wealth Information,” *The Economic Journal*, 113(488), 282–301.
- CAGETTI, M. (2003): “Wealth Accumulation Over the Life Cycle and Precautionary Savings,” *Journal of Business and Economic Statistics*, 21(3), 339–353.
- CARROLL, C. D. (1997): “Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis,” *The Quarterly Journal of Economics*, 112(1), pp. 1–55.
- (2001): “Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation),” *Advances in Macroeconomics*, 1(1), Article 6.
- (2006): “The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems,” *Economics Letters*, 91(3), 312–320.
- CARROLL, C. D. AND A. A. SAMWICK (1997): “The nature of precautionary wealth,” *Journal of Monetary Economics*, 40(1), 41–71.
- DYNAN, K. E. (2000): “Habit Formation in Consumer Preferences: Evidence from Panel Data,” *American Economic Review*, 90(3), 391–406.
- FERNÁNDEZ-VILLAYERDE, J. AND D. KRUEGER (2007): “Consumption over the Life Cycle: Facts from Consumer Expenditure Survey Data,” *The Review of Economics and Statistics*, 89(3), 552–565.
- FERNÁNDEZ-VILLAYERDE, J., J. F. RUBIO-RAMÍREZ AND M. S. SANTOS (2006): “Convergence Properties of the Likelihood of Computed Dynamic Models,” *Econometrica*, 74(1), 93–119.
- GOURINCHAS, P.-O. AND J. A. PARKER (2002): “Consumption Over the Life Cycle,” *Econometrica*, 70(1), 47–89.
- HALL, R. E. AND F. S. MISHKIN (1982): “The Sensitivity of Consumption to Transitory Income,” *Econometrica*, 50(2), 461–481.
- HAMILTON, J. D. (1994): *Time Series Analysis*. Princeton University Press, Princeton, New Jersey.
- HEATHCOTE, J., K. STORESLETTEN AND G. L. VIOLANTE (forthcoming): “Consumption and Labor Supply with Partial Insurance: An Analytical Framework,” *American Economic Review*.
- JØRGENSEN, M. (2007): “Danskernes Pensionsopsparinger,” Discussion paper, SFI.
- JØRGENSEN, T. H. (2013): “Structural estimation of continuous choice models: Evaluating the EGM and MPEC,” *Economics Letters*, 119(3), 287–290.
- LAWRANCE, E. C. (1991): “Poverty and the Rate of Time Preference: Evidence from Panel Data,” *Journal of Political Economy*, 99(1), 54–77.
- LETH-PETERSEN, S. (2010): “Intertemporal Consumption and Credit Constraints: Does Total Expenditure Respond to an Exogenous Shock to Credit?,” *American Economic Review*, 100(3), 1080–1103.

- MAREKWICA, M., A. SCHAEFER AND S. SEBASTIAN (2013): “Life cycle asset allocation in the presence of housing and tax-deferred investing,” *Journal of Economic Dynamics and Control*, 37(6), 1110 – 1125.
- MEGHIR, C. AND L. PISTAFERRI (2004): “Income Variance Dynamics and Heterogeneity,” *Econometrica*, 72(1), 1–32.
- MINISTRY OF FINANCE (2003): *ældres sociale vilkår*. (In Danish).
- MOON, H. R. AND F. SCHORFHEIDE (2009): “Estimation with overidentifying inequality moment conditions,” *Journal of Econometrics*, 153(2), 136 – 154.
- RENDAHL, P. (forthcoming): “Inequality Constraints and Euler Equation Based Solution Methods,” *The Economic Journal*.
- RUNKLE, D. E. (1991): “Liquidity constraints and the permanent-income hypothesis: Evidence from panel data,” *Journal of Monetary Economics*, 27(1), 73 – 98.
- RUST, J. (1987): “Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher,” *Econometrica*, 55(5), 999–1033.
- STORESLETTEN, K., C. I. TELMER AND A. YARON (2004): “Consumption and risk sharing over the life cycle,” *Journal of Monetary Economics*, 51(3), 609 – 633.
- SU, C.-L. AND K. L. JUDD (2012): “Constrained Optimization Approaches to Estimation of Structural Models,” *Econometrica*, 80(5), 2213–2230.
- THUROW, L. C. (1969): “The Optimum Lifetime Distribution of Consumption Expenditures,” *The American Economic Review*, 59(3), pp. 324–330.
- VAN DER KLAUW, W. AND K. I. WOLPIN (2008): “Social security and the retirement and savings behavior of low-income households,” *Journal of Econometrics*, 145(1-2), 21–42.
- WOOLDRIDGE, J. M. (2002): *Econometric Analysis of Cross Section and Panel Data*. Massachusetts Institute of Technology, USA.
- YANG, F. (2009): “Consumption over the life cycle: How different is housing?,” *Review of Economic Dynamics*, 12(3), 423 – 443.

A Additional Figures and Tables

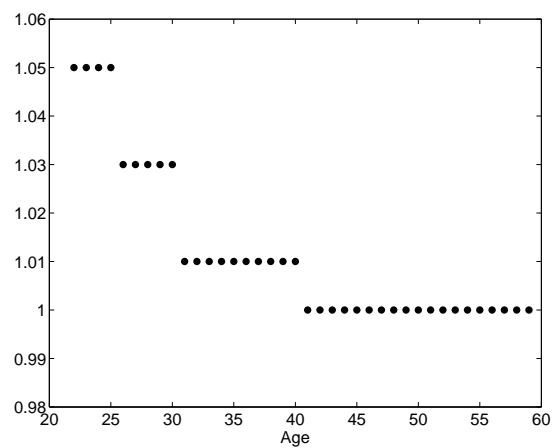


Figure A.1 – Income Growth Profile used in Simulations.

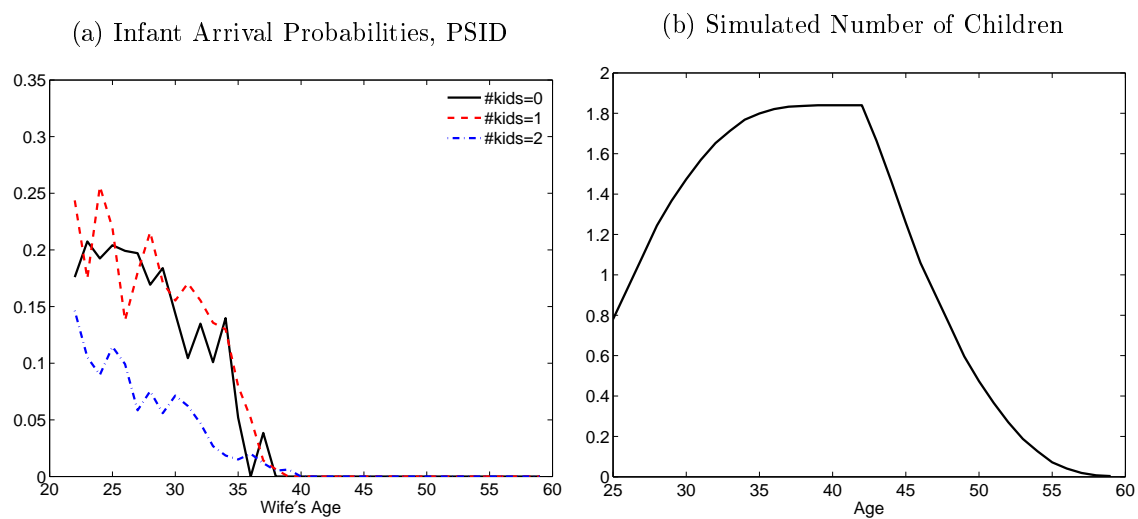


Figure A.2 – Estimated Arrival Probability of Infant (PSID) and Simulated Number of Children.

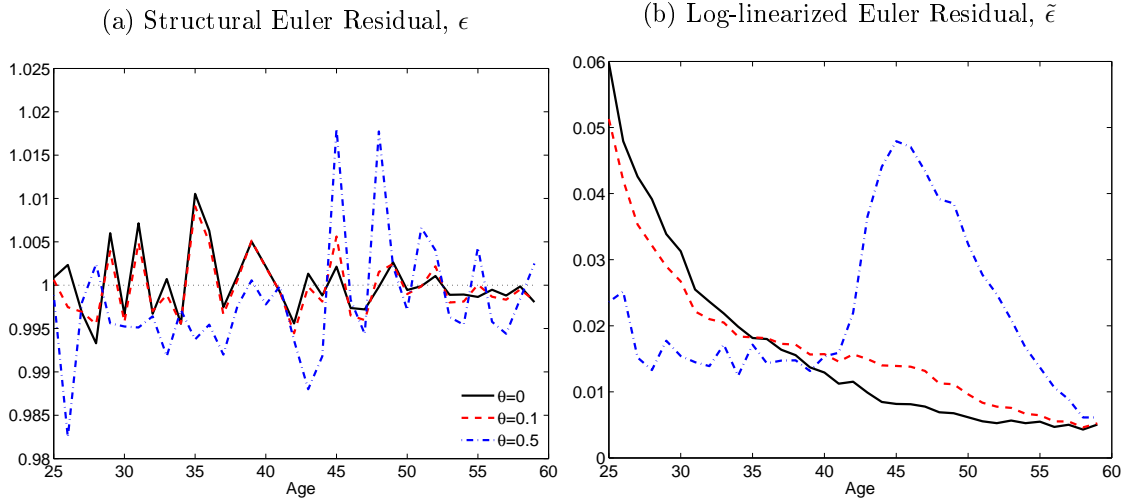


Figure A.3 – Euler Residuals, Model Without “Explicit” Credit Constraint, but $\varphi = 0.003$, $\mu = 0$.

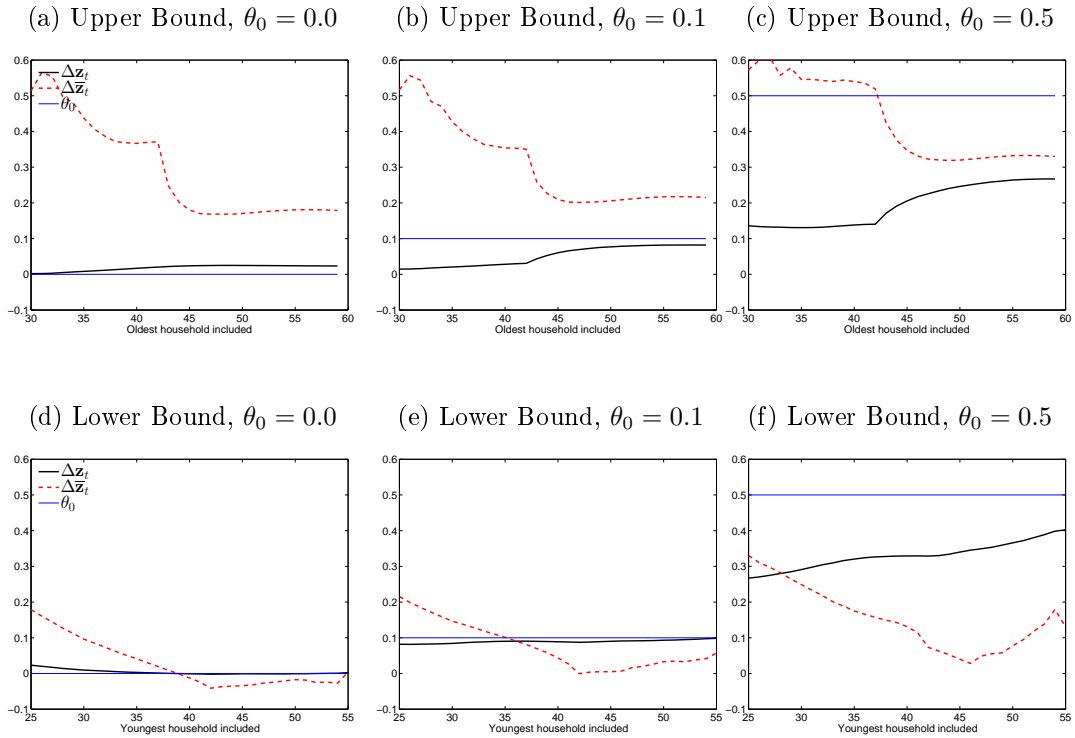


Figure A.4 – Lower and Upper Bounds, Log-linear Euler. Model Without “Explicit” Credit Constraint, but $\varphi = 0.003$, $\mu = 0$.

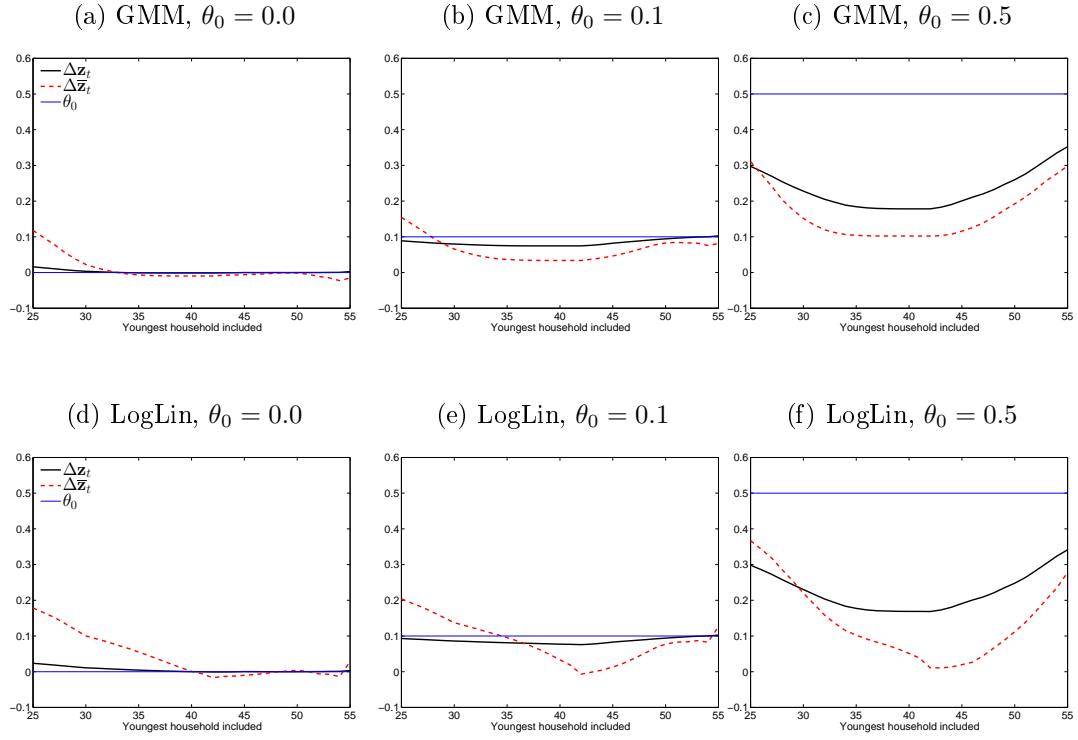


Figure A.5 – Estimation on Sub-samples, from age 59 through 25. Deterministic Model.

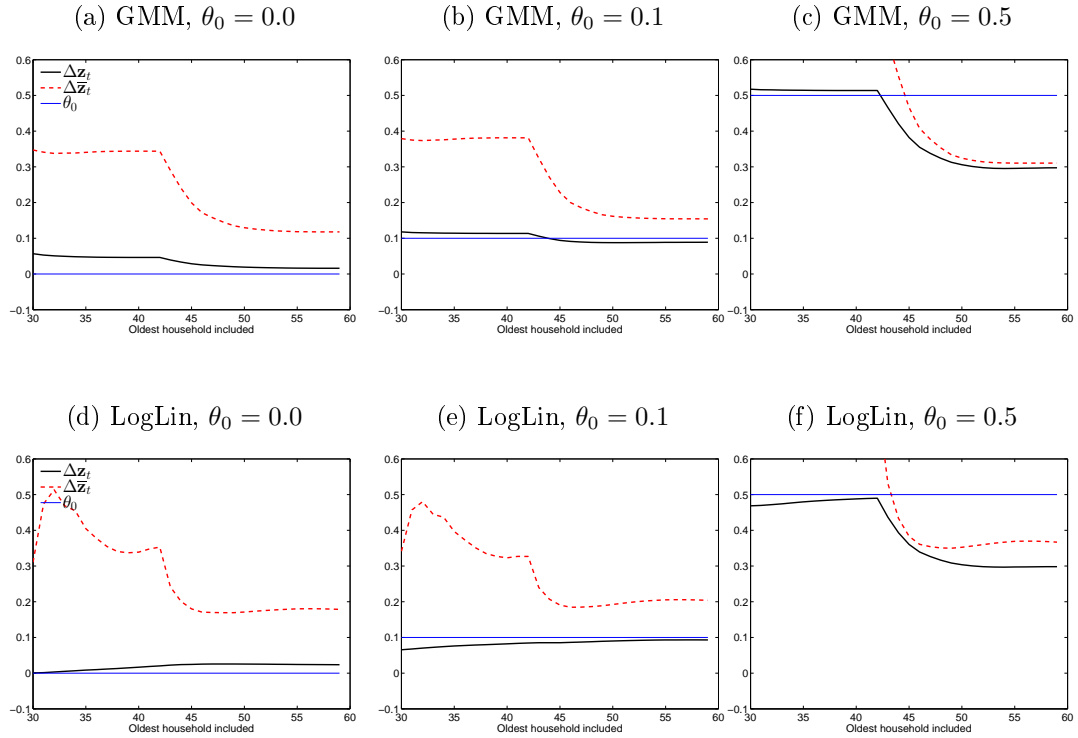


Figure A.6 – Estimation on Sub-samples, from age 25 through 59. Deterministic Model.

Table A1 – Sample Selection Criteria, Danish Data.

	Household-time observations	Share of original sample
Original, all couples	16,268,110	1.000
Wife age	8,832,011	0.543
Husband age	5,874,962	0.361
No information on education	5,848,884	0.360
No more than 3 children	5,716,046	0.351
Both in labor force	2,238,076	0.138
No negative disposable income	2,231,204	0.137
Child outside allowed range	2,230,326	0.137
Extreme observations	1,522,994	0.094
Resources missing	1,281,952	0.079

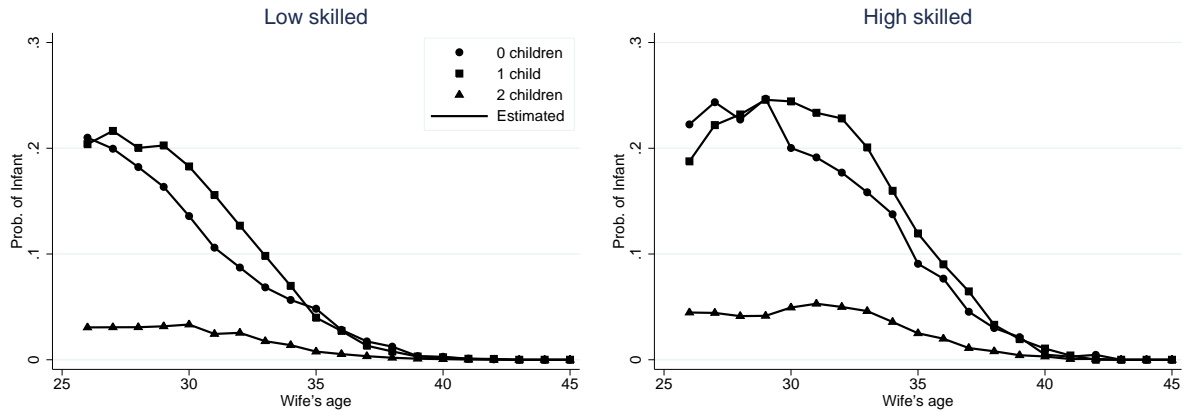


Figure A.7 – Actual and Predicted Arrival of Infant. Female Age profiles for Low and High skilled Households. Danish Data.

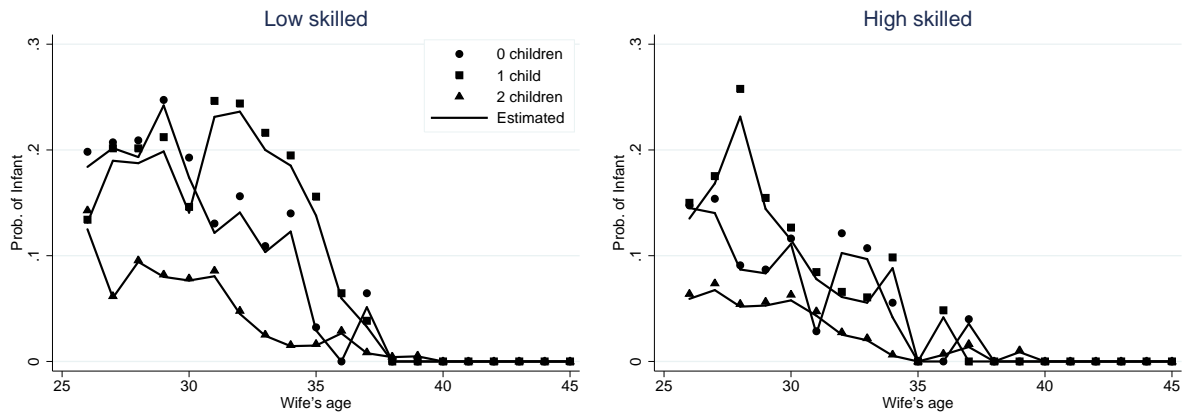


Figure A.8 – Actual and Predicted Arrival of Infant. Female Age profiles for Low and High skilled Households. PSID.

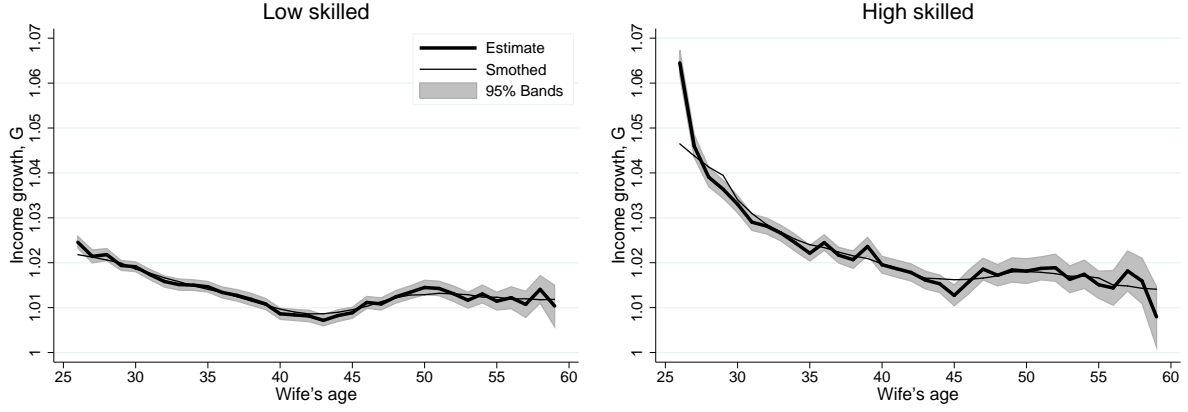


Figure A.9 – Age Profile of Income Growth, \hat{G}_t , Danish Data.

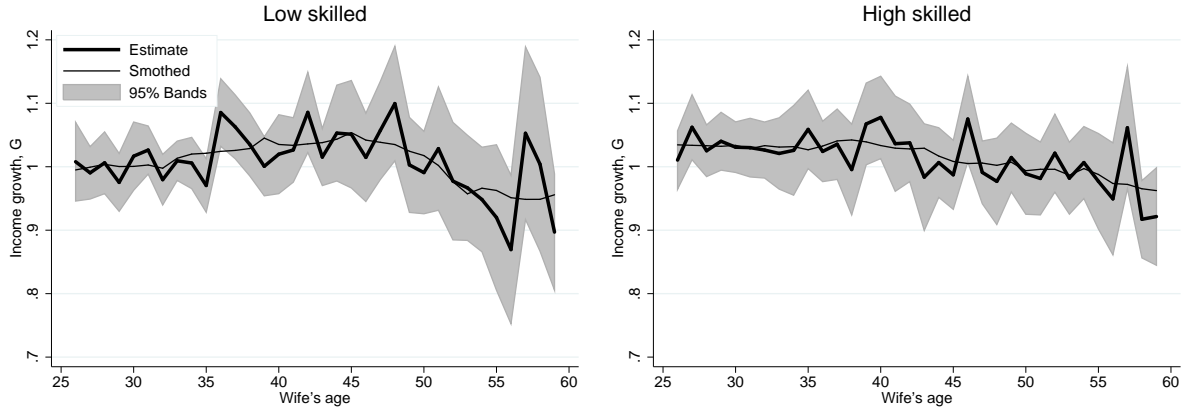


Figure A.10 – Age Profile of Income Growth, \hat{G}_t , PSID.

Table A2 – Monte Carlo Results, Structural Estimation.

	$\theta_0 = 0.0$	$\theta_0 = 0.1$	$\theta_0 = 0.3$	$\theta_0 = 0.5$
$N = 1000, T = 5$	0.004 (0.034)	0.115 (0.062)	0.300 (0.099)	0.504 (0.080)
$N = 1000, T = 20$	-0.000 (0.017)	0.106 (0.039)	0.307 (0.078)	0.510 (0.057)
$N = 50000, T = 5$	0.000 (0.005)	0.099 (0.010)	0.299 (0.017)	0.501 (0.013)
$N = 50000, T = 20$	0.000 (0.002)	0.100 (0.004)	0.301 (0.010)	0.502 (0.009)

Notes: Means and standard deviations (in parenthesis) are reported. All results are based on 50 independent estimations on simulated data from the model described in Section 3 with the parameters presented in Table 2. See footnote to Table 3.

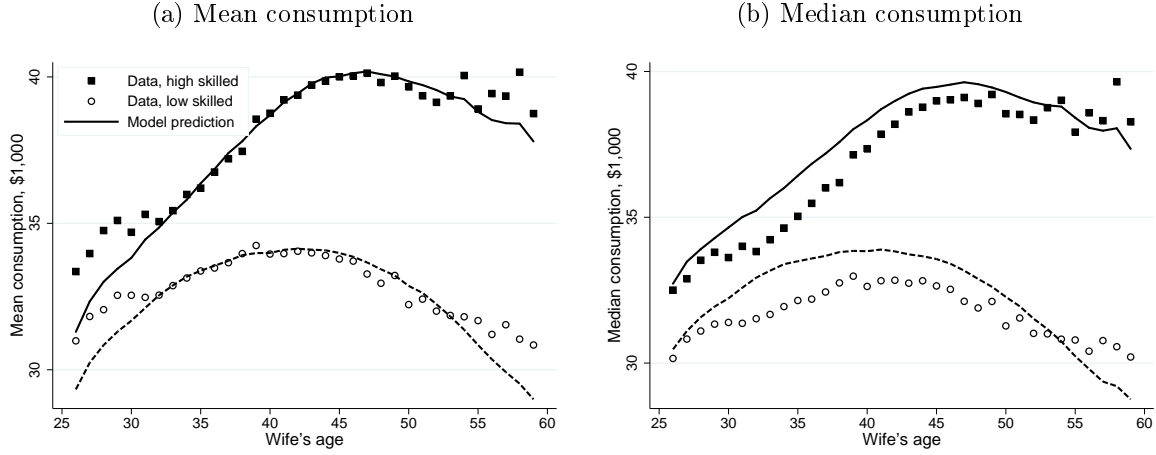


Figure A.11 – Actual and Predicted Consumption profiles, Danish Data.

Table A3 – Robustness Check: Alternative Objective Functions.

	Danish Registers				PSID			
	Low skilled		High skilled		Low skilled		High skilled	
	Logistic	Abs	Logistic	Abs	Logistic	Abs	Logistic	Abs
ρ	1.565 (0.076)	0.588	1.986 (0.054)	1.245	1.272 (0.219)	1.148	1.821 NA	1.956
β	0.973 (0.000)	0.974	0.981 (0.000)	0.982	0.976 (0.002)	0.982	0.950 NA	0.960
γ^\dagger	1.152 (0.013)	1.010	1.013 (0.010)	0.921	1.300 (1.300)	1.300	1.300 (1.300)	1.300
σ_ξ	0.421 (0.000)		0.444 (0.001)		0.483 (0.005)		0.626 (0.007)	
θ_{11}	0.001 (0.004)	0.001	0.012 (0.007)	0.010	0.201 (0.091)	0.181	0.139 (0.149)	0.309
θ_{12}	-0.008 (0.002)	-0.002	0.021 (0.005)	0.015	0.269 (0.085)	0.172	0.372 (0.121)	0.385
θ_{21}	-0.010 (0.003)	-0.003	0.003 (0.005)	0.004	0.057 (0.062)	0.044	0.068 (0.120)	-0.018
θ_{22}	0.006 (0.003)	0.003	0.012 (0.005)	0.009	0.075 (0.082)	0.099	0.205 (0.126)	0.175
θ_{31}	0.006 (0.005)	0.005	0.028 (0.008)	0.015	0.258 (0.116)	0.234	0.143 (0.141)	0.236
θ_{32}	0.014 (0.007)	0.006	0.018 (0.011)	0.004	0.092 (0.131)	0.098	0.562 (0.262)	0.661

Notes: Standard errors are based on the inverse of the hessian. "NA" refers to Not Available and stems from the hessian being close to singular and the resulting parameter variances are negative. This indicates that the objective function is flat within numerical precision and the estimates and standard errors should be interpreted with caution. "Logistic" refers to results assuming additive Logistically distributed measurement error in normalized consumption. "Abs" refers to results from minimizing the absolute difference between observed and predicted consumption. Standard errors are not reported since these should be bootstrapped due to the non-differentiability of objective function at zero.

[†] For US households, using the PSID, the retirement motive is fixed across educational groups to be 1.3 based on results for Danish households.

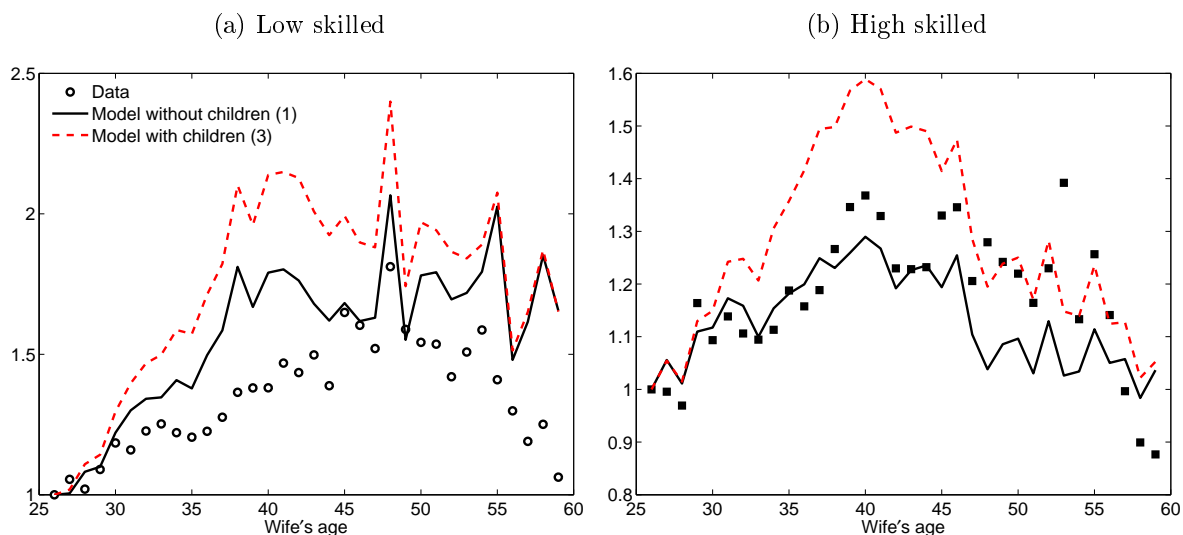


Figure A.12 – Actual and Predicted Consumption profiles Relative to age 26, PSID.

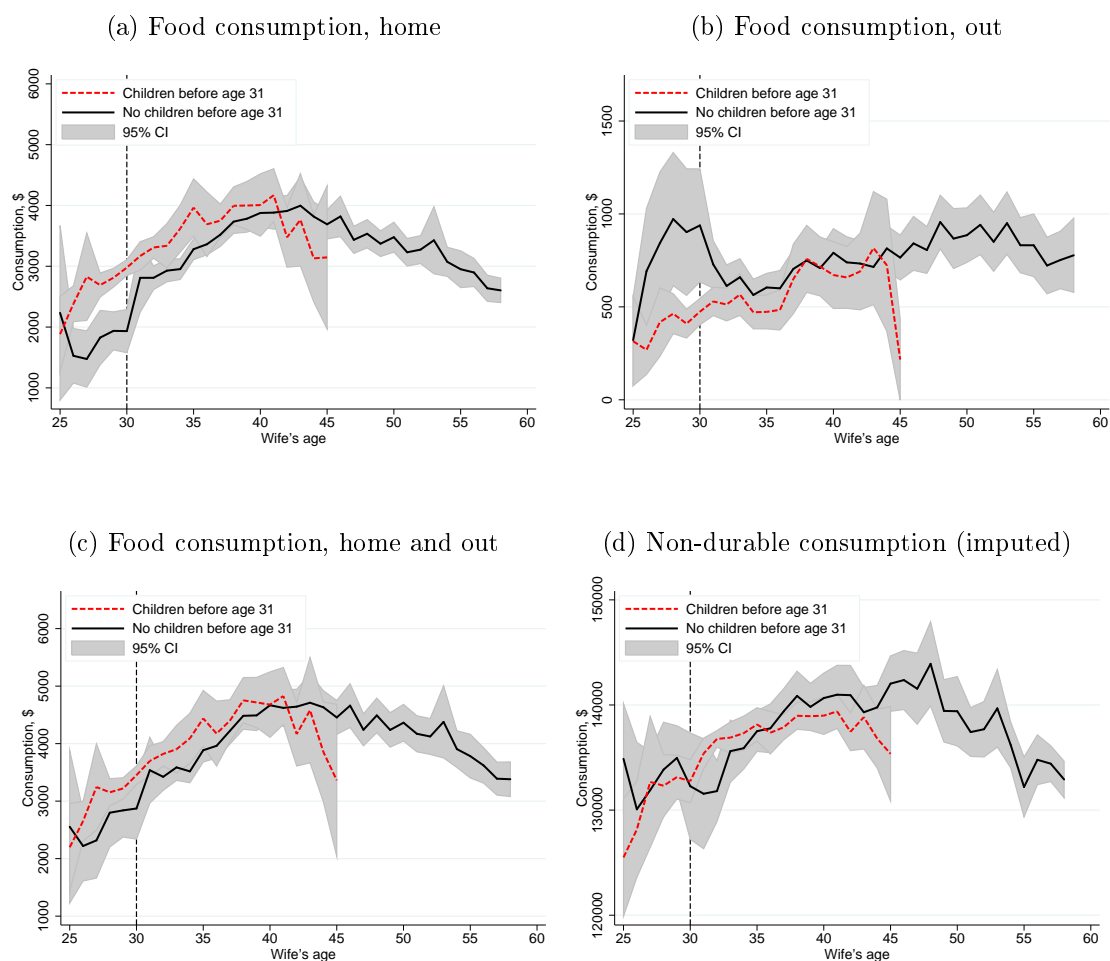


Figure A.13 – Consumption Components for Households With and Without Children, PSID.

Table A4 – Estimated Preference Parameters, PSID. Deterministic Arrival of Children.

		Low skilled			High skilled		
		(1)	(2)	(3)	(1)	(2)	(3)
ρ	Risk aversion	1.829 NA	1.702 (1.096)	1.605 NA	1.828 (0.662)	1.953 (0.785)	1.431 NA
β	Discount factor	0.954 NA	0.965 (0.021)	0.963 (0.005)	0.931 (0.027)	0.939 (0.047)	0.948 NA
γ^\dagger	Retirement	1.300	1.300	1.300	1.300	1.300	1.300
σ_ξ	Meas, err.	1.343 (0.011)	1.355 (0.011)	1.355 (0.011)	3.582 (0.029)	3.538 (0.030)	3.537 (0.030)
Taste shifter, $v(\mathbf{z}; \theta) = \exp(\theta' \mathbf{z})$							
θ	# of children	0.179 (0.118)			0.271 (0.328)		
Taste shifter, $v(\mathbf{z}; \theta) = 1 + \theta' \mathbf{z}$							
θ_{11}	1. child ≤ 10	−0.016 (0.173)			−0.141 NA		
θ_{12}	1. child > 10	0.186 NA			0.149 NA		
θ_{21}	2. child ≤ 10	0.154 NA			0.144 (0.408)		
θ_{22}	2. child > 10	0.056 NA			0.308 NA		
θ_{31}	3. child ≤ 10	0.337 NA			0.070 (0.470)		
θ_{32}	3. child > 10	0.149 NA			0.348 (0.462)		
$-\mathcal{L}(\Theta)$		0.83295	0.82087	0.82083	1.13714	1.09120	1.09117
LR [p -val]			353.3[0.00]	354.3[0.00]		1612.1[0.00]	1613.1[0.00]
# of observations		8,333	8,333	8,333	8,672	8,672	8,672

Notes: Standard errors are based on the inverse of the hessian. "NA" refers to Not Available and stems from the hessian being close to singular and the resulting parameter variances are negative. This indicates that the objective function is flat within numerical precision and the estimates and standard errors should be interpreted with caution.

[†] The retirement motive is fixed across educational groups to be 1.3.

Table A5 – Robustness Checks: Income Variance and Heterogeneous Measurement Error, PSID.

	Probabilistic Arrival of Children				Deterministic Arrival of Children			
	Low skilled		High skilled		Low skilled		High skilled	
	G&P	Het.	G&P	Het.	G&P	Het.	G&P	Het.
ρ	1.596 NA	2.411	1.495 (0.020)	1.930	1.605 NA	2.459	1.431 NA	1.903
β^\dagger	0.966 NA	0.950	0.948 (0.013)	0.950	0.963 (0.005)	0.950	0.948 NA	0.950
γ^\dagger	1.300	1.300	1.300	1.300	1.300	1.300	1.300	1.300
σ_ξ	1.342 (0.011)		3.582 (0.029)		1.355 (0.011)		3.537 (0.030)	
θ_{11}	0.117 (0.288)	-0.355	-0.087 (0.549)	0.115	-0.016 (0.173)	-0.139	-0.141 NA	0.046
θ_{12}	0.353 (0.292)	-0.362	0.247 (0.530)	0.078	0.186 NA	-0.131	0.149 NA	0.021
θ_{21}	0.158 (0.221)	-0.084	0.158 (0.521)	-0.054	0.154 NA	-0.242	0.144 (0.408)	-0.126
θ_{22}	0.023 (0.313)	0.017	0.320 (0.653)	0.285	0.056 NA	-0.155	0.308 NA	-0.103
θ_{31}	0.535 NA	-0.183	-0.016 (0.676)	-0.241	0.337 NA	-0.118	0.070 (0.470)	-0.082
θ_{32}	0.277 (0.483)	-0.119	0.630 (0.369)	0.087	0.149 NA	-0.158	0.348 (0.462)	0.029

Notes: Standard errors are based on the inverse of the hessian. "NA" refers to Not Available and stems from the hessian being close to singular and the resulting parameter variances are negative. This indicates that the objective function is flat within numerical precision and the estimates and standard errors should be interpreted with caution. "G&P" refers to a model in which the permanent and transitory income variances are calibrated to those found in [Gourinchas and Parker \(2002\)](#), $\sigma_\eta = 0.0212$ and $\sigma_\varepsilon = 0.0440$ respectively. "Het." refers to heterogeneous multiplicative measurement error in consumption, as in Example 2 in Section 5.

[†] The retirement motive is fixed across educational groups to be 1.3.

[‡] The discount factor, β is fixed at 0.95 when allowing for heterogeneous multiplicative measurement error. Using first differences in log consumption effectively only utilize information on households with at least two consecutive periods. The reduced number of observations in effect made joint identification of ρ and β impossible.

B Solving the Four-Period Model

All variables are normalized with income. Hence, e.g., $m_1 = (A_0 + Y_1)/Y_1 = G_1^{-1}a_0 + 1$ since $Y_1 = G_1Y_0$. In all other periods, income is constant. This normalization facilitates solving the model analytically for *all* possible values of income. The resulting consumption function should be multiplied with current period income to give the unnormalized level of consumption, $C_t^* = Y_t c_t^*$. The consumption functions in periods one, two and three are independent of whether children arrive deterministically or probabilistically, since it is assumed that children, if present in period one, will move with certainty in period two. Therefore, I first solve for optimal consumption in period three, two and one and then turn to the initial period consumption, prior to potential arrival of children. This analysis is split between the model in which children arrive deterministically and the model in which children arrive probabilistically.

In the terminal period, period three, all resources are consumed ($c_3^* = m_3$) and the *unconstrained* Euler equation linking period two and period three consumption is then

$$c_2^{-\rho} = m_3^{-\rho}$$

such that inserting normalized resources, $m_3 = m_2 - c_2 + 1$ and re-arranging shows that optimal consumption in period two is given by,

$$c_2^*(m_2) = \min \left\{ m_2, \frac{1}{2}(m_2 + 1) \right\}, \quad (10)$$

where I have used that consumption cannot exceed available resources. In period one, a child may be present, such that the *unconstrained* Euler equation is given by

$$c_1^{-\rho} \exp(\theta \mathbf{z}_1) = c_2^{-\rho},$$

such that inserting normalized resources and re-arranging yields,

$$c_1^*(m_1 | \mathbf{z}_1) = \min \left\{ m_1, \frac{m_1 + 2}{1 + 2 \exp(-\rho^{-1} \theta \mathbf{z}_1)} \right\}, \quad (11)$$

where I have used that only if nothing is saved in period one ($c_1^*(m_1 | \mathbf{z}_1) = m_1$) will consumption on period two equal available resources. Since income is constant across period one and two (and three) the Euler equation linking period one and period two consumption is valid as long as period one consumption is less than the available resources ($c_1^*(m_1 | \mathbf{z}_1) < m_1$) and else $c_1^*(m_1 | \mathbf{z}_1) = m_1$ and $c_1^*(m_2) = m_2$. Intuitively, if the constraint is not binding in period one, in which children might be present, it will certainly not be binding in the subsequent period.

Optimal consumption in the initial period depends on whether children arrive deterministically or probabilistically in period one. I first derive optimal consumption in the case where children arrive deterministically and then turn to the probabilistic arrival of children.

B.1 Initial Period Consumption: Deterministic Arrival of Children

In the first period, the *unconstrained* Euler equation is

$$c_0^{-\rho} = G_1^{-\rho} \exp(\theta \mathbf{z}_1) c_1^{-\rho},$$

since income grows with a factor G_1 from period zero to period one. Since consumption in period one is potentially constrained, this has to be explicitly taken into account. First, assuming that period one consumption is less than available resources, $c_1 < m_1$, inserting the optimal consumption found in (11) and tedious re-arranging yields optimal consumption in this case,

$$c_0^*(m_0 | \mathbf{z}_1)^{\text{det}}|_{c_1 < m_1} = \frac{m_0 + 3G_1}{3 + \exp(\rho^{-1}\theta \mathbf{z}_1)}. \quad (12)$$

If, on the other hand, consumption in period one is constrained ($c_1 = m_1$), inserting this in the Euler equation and re-arranging yields,

$$c_0^*(m_0 | \mathbf{z}_1)^{\text{det}}|_{c_1 = m_1} = \frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta \mathbf{z}_1)}. \quad (13)$$

To determine which of the consumption functions is relevant, note that equation (12) would imply a too high level of consumption in period zero if ignoring, that at some point, consumption in period one would be constrained because “too little” is saved in period zero. Hence,

$$\tilde{c}_0^*(m_0 | \mathbf{z}_1)^{\text{det}} = \min \left\{ m_0, \frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta \mathbf{z}_1)}, \frac{m_0 + 3G_1}{3 + \exp(\rho^{-1}\theta \mathbf{z}_1)} \right\},$$

where the level of period zero resources implying that consumption in period one is constrained is the level of resources, $\underline{m}_0 \leq \exp(\rho^{-1}\theta \mathbf{z}_1)G_1$, that solves,

$$\frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta \mathbf{z}_1)} \leq \frac{m_0 + 3G_1}{3 + \exp(\rho^{-1}\theta \mathbf{z}_1)},$$

such that when $m_0 \leq \exp(\rho^{-1}\theta \mathbf{z}_1)G_1$ optimal consumption in period zero is given by equation (13) and when $m_0 > \exp(\rho^{-1}\theta \mathbf{z}_1)G_1$ optimal consumption in period zero is given by equation (12).

When households are initiated with zero wealth ($a_{-1} = 0$) available normalized resources in period zero is one, $m_0 = 1$, and only equation (13) is relevant. This is because $m_0 = 1 \leq \exp(\rho^{-1}\theta \mathbf{z}_1)G_1$ for all values of $\theta \geq 0$ and $G_1 \geq 1$. Therefore, assuming no initial wealth and deterministic arrival of children, optimal consumption in period zero is given by

$$c_0^*(m_0 | \mathbf{z}_1)^{\text{det}} = \min \left\{ m_0, \frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta \mathbf{z}_1)} \right\}. \quad (14)$$

B.2 Initial Period Consumption: Probabilistic Arrival of Children

The analysis, if children arrive probabilistically, is slightly more complicated than the above where children arrive deterministically. The *unconstrained* Euler equation is here given by

$$c_0^{-\rho} = G_1^{-\rho} c_1^{-\rho} (p \exp(\theta) + 1 - p),$$

such that in case where period one consumption is unconstrained ($c_1 < m_1$), inserting optimal consumption from equation (11) and re-arranging yields

$$c_0^*(m_0)|_{c_1 < m_1} = \frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}}. \quad (15)$$

However, if households are potentially credit constrained if a child arrives next period, the model has in general no analytical solution because the Euler equation is

$$\begin{aligned} c_0^{-\rho} &= G_1^{-\rho} \left[c_1^{-\rho} (1-p) + p \exp(\theta) m_1^{-\rho} \right], \\ &= G_1^{-\rho} \left[\frac{1}{3} (G_1^{-1} (m_0 - c_0) + 3)^{-\rho} (1-p) + p \exp(\theta) [G_1^{-1} (m_0 - c_0) + 1]^{-\rho} \right], \end{aligned}$$

with no general analytical solution for c_0 . To complete arguments, I solve for the optimal consumption numerically using the EGM proposed by [Carroll \(2006\)](#), and use that solution, denoted $c_0^*(m_0)|_{c_1=m_1}^{\text{num}}$. We have that optimal period zero consumption is given by

$$c_0^*(m_0) = \min \left\{ m_0, c_0^*(m_0)|_{c_1=m_1}^{\text{num}}, \frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}} \right\}, \quad (16)$$

where Figure B.1a reports the consumption function in the stochastic case for the baseline parameters used herein ($p = 0.5$, $\rho = 2$, and $\theta = 0.5$). Figure B.1b reports the consumption function in the deterministic case. The numerical solution is also reported to confirm the results.

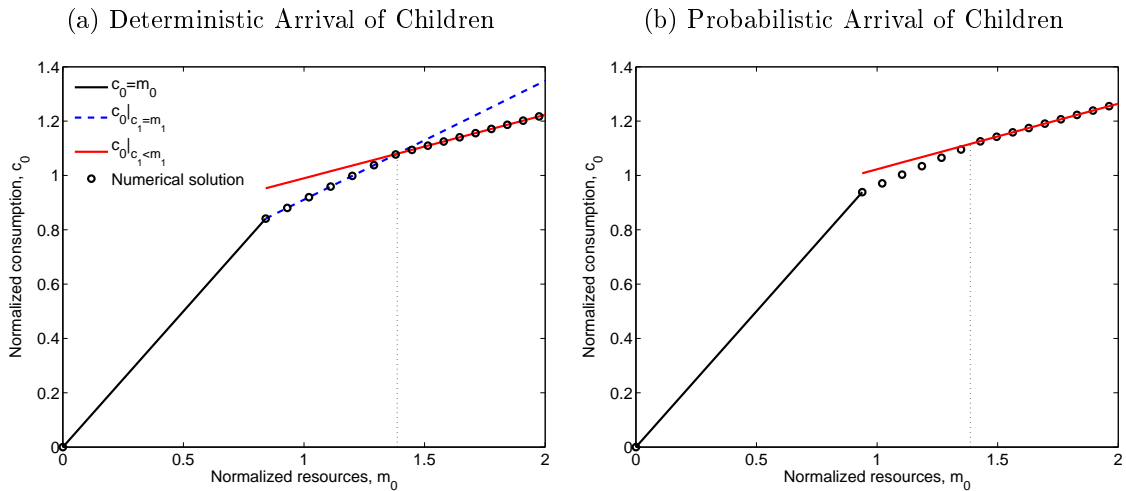


Figure B.1 – Period Zero Optimal Consumption Functions.

B.3 Small Upwards Bias of IV using Young Households Without Credit Constraints

Here, I show that in the model where children arrive probabilistically, when the effect of children on consumption is large enough to effectively dominate the credit constraint, there is only a small positive bias from IV estimation. The IV estimator is

$$\frac{\mathbb{E}[\Delta \log C'_1 Z]}{\mathbb{E}[Z'Z]} = \frac{1}{p}(p\Delta \log(C_1)|_{\mathbf{z}_1=1} + (1-p)\Delta \log(C_1)|_{\mathbf{z}_1=0}),$$

such that inserting optimal consumption in absence of credit constraints,

$$C_0 = Y_0 \frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}}, \quad C_1(\mathbf{z}_1) = Y_1 \frac{m_1 + 2}{1 + 2\exp(-\rho^{-1}\theta \mathbf{z}_1)},$$

yields

$$\begin{aligned} \frac{\mathbb{E}[\Delta \log C'_1 Z]}{\mathbb{E}[Z'Z]} &= \log \left(Y_1 \frac{m_1 + 2}{1 + 2\exp(-\rho^{-1}\theta)} \right) - \log \left(Y_0 \frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}} \right) \\ &+ (1-p)/p \left(\log \left(Y_1 \frac{1}{3}(m_1 + 2) \right) - \log \left(Y_0 \frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}} \right) \right) \\ &= ((1 + (1-p)/p)) \log(m_1 + 2) + \log \left(\frac{\exp(\rho^{-1}\theta)}{\exp(\rho^{-1}\theta) + 2} \right) - (1-p)/p \log(3) + p^{-1} \log(G_1) \\ &- (1 + (1-p)/p) \log \left(\frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}} \right) \\ &= \rho^{-1}\theta + p^{-1} \log(G_1) - (\log(\exp(\rho^{-1}\theta) + 2) + (1-p)/p \log(3)) \\ &+ p^{-1} \left[\log(m_1 + 2) - \log \left(\frac{m_0 + 3G_1}{1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}}} \right) \right] \\ &= \rho^{-1}\theta - (\log(\exp(\rho^{-1}\theta) + 2) + (1-p)/p \log(3)) \\ &+ p^{-1} \left[\log \left(G_1 \frac{m_1 + 2}{m_0 + 3G_1} \right) + \log \left(1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}} \right) \right] \\ &= \rho^{-1}\theta - (\log(\exp(\rho^{-1}\theta) + 2) + (1-p)/p \log(3)) \\ &+ p^{-1} \left[\log \left(\frac{m_0 + 3G_1 - c_0}{m_0 + 3G_1} \right) + \log \left(1 + [p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho]^{\frac{1}{\rho}} \right) \right], \end{aligned}$$

where inserting again c_0 from equation (15) and re-arranging finally gives the IV estimator as $\rho^{-1}\theta + \omega$, where

$$\omega \equiv p^{-1} [\rho^{-1} \log(p(\exp(\rho^{-1}\theta) + 2)^\rho + (1-p)3^\rho) - (p \log(\exp(\rho^{-1}\theta) + 2) + (1-p) \log(3))] \geq 0,$$

such that defining $\omega_1 \equiv (\exp(\rho^{-1}\theta) + 2)^\rho$ and $\omega_2 \equiv 3^\rho$, the bias of the IV estimator can be seen to be the difference in the log-expected value and the expected log value;

$$\omega = p^{-1} \rho^{-1} (\log(p\omega_1 + (1-p)\omega_2) - (p \log \omega_1 + (1-p) \log \omega_2)),$$

which is always positive since the logarithm is a concave function.

C Solving the Baseline Structural Model

To reduce the number of state variables, all relations are normalized by permanent income, P_t , and small letter variables denote normalized quantities (e.g., $c_t = C_t/P_t$). The model is solved recursively by backwards induction, starting with the terminal period, T . Within a given period, optimal consumption is found using the Endogenous Grid Method (EGM) by [Carroll \(2006\)](#).

The EGM constructs a grid over end-of-period wealth, a_t , rather than beginning-of-period resources, m_t . Denote this grid of Q points as $\hat{a}_t = (\underline{a}_t, a_t^1, \dots, a_t^{Q-1})$ in which \underline{a}_t is a lower bound on end-of-period wealth that I will discuss in great detail below. The endogenous level of beginning-of-period resources consistent with end-of-period assets, \hat{a}_t , and optimal consumption, c_t^* , is given by $m_t = \hat{a}_t + c_t^*(m_t, \mathbf{z}_t)$.

In the terminal period, independent of the presence of children, households consume all their remaining wealth, $c_T = m_T$. In preceding periods, in which households are retired, consumption across periods satisfy the Euler equation

$$u'(c_t) = \max \left\{ u'(m_t), R\beta \frac{v(\mathbf{z}_{t+1}; \theta)}{v(\mathbf{z}_t; \theta)} u'(c_{t+1}) \right\}, \quad \forall t \in [T_r, T],$$

where consumption cannot exceed available resources. When retired, households do not produce new offspring and the age of children (\mathbf{z}_t) evolves deterministically.

The normalized consumption Euler equation in periods prior to retirement is given by

$$u'(c_t) = \max \left\{ u'(m_t + \kappa), R\beta \mathbb{E}_t \left[\frac{v(\mathbf{z}_{t+1}; \theta)}{v(\mathbf{z}_t; \theta)} u'(c_{t+1} G_{t+1} \eta_{t+1}) \right] \right\}, \quad \forall t < T_r,$$

such that when $\hat{a}_t > -\kappa$ optimal consumption can be found by inverting the Euler equation

$$c_t^*(m_t, \mathbf{z}_t) = \left(\beta R \mathbb{E}_t \left[\frac{v(\mathbf{z}_{t+1}; \theta)}{v(\mathbf{z}_t; \theta)} (G_{t+1} \eta_{t+1})^{-\rho} \underbrace{\check{c}_{t+1}^*((G_{t+1} \eta_{t+1})^{-1} R \hat{a}_t + \varepsilon_{t+1}, \mathbf{z}_{t+1})^{-\rho}}_{=m_{t+1}} \right] \right)^{-\frac{1}{\rho}},$$

where $\check{c}_{t+1}^*(m_{t+1}, \mathbf{z}_{t+1})$ is a linear interpolation function of optimal consumption next period, found in the last iteration. Since \hat{a}_t is the constructed grid, it is trivial to determine in which regions the credit constraint is binding and not. I will discuss this in detail below.

The expectations are over next period arrival of children (\mathbf{z}_{t+1}) and transitory (ε_{t+1}) and permanent income shocks (η_{t+1}). Eight Gauss-Hermite quadrature points are used for each income shock to approximate expectations. $Q = 80$ discrete grid points are used in \hat{a}_t to approximate the consumption function with more mass at lower levels of wealth to approximate accurately the curvature of the consumption function. The number of points was chosen such that the change in the optimized log likelihood did not change significantly, as proposed in [Fernández-Villaverde, Rubio-Ramírez and Santos \(2006\)](#).

The arrival probability of a child next period is a function of the wife's age and number of children today, $\pi_{t+1}(\mathbf{z}_t)$. No more than three children can live inside a household at a given point in time and infants cannot arrive when the household is older than 43. The next period's state is therefore calculated by increasing the age of children by one and if the age is 21, the child moves. In principle, there is $22^3 = 10,648$ combinations three children can be either not

present (NC) or aged zero through 20. To reduce computation time, children are organized such that child one is the oldest at all times, the second child is the second oldest and child three is the youngest child. To illustrate, imagine a household which in period t has, say, two children aged 20 and 17, $\mathbf{z}_t = (\text{age}_{1,t} = 20, \text{age}_{2,t} = 17, \text{age}_{3,t} = \text{NC})$, then, in period $t+1$, only one child will be present; $\mathbf{z}_{t+1} = (\text{age}_{1,t+1} = 18, \text{age}_{2,t+1} = \text{NC}, \text{age}_{3,t+1} = \text{NC})$, given no new offspring arrives. Had new offspring arrived, then $\text{age}_{2,t+1} = 0$.

C.1 Credit Constraint and Utility Induced Constraints

Since the EGM works with end-of-period wealth rather than beginning-of-period resources, credit constraints can easily be implemented by adjusting the lowest point in the grid, \underline{a}_t . The potentially binding credit constraint next period is implemented by the rule, $c_{t+1}^* = m_{t+1}$ if m_{t+1} is lower than some threshold level, \mathbf{m}_{t+1}^* . Including the credit constraint as the lowest point, $\underline{a}_{t+1} = -\kappa$, the lowest level of resources endogenously determined in the last iteration, \underline{m}_{t+1} , is the exact level of resources where households are on the cusp of being credit constrained, i.e., $\mathbf{m}_{t+1}^* = \underline{m}_{t+1}$. This ensures a very accurate interpolation and requires no additional handling of shadow prices of resources in the constrained Euler equation, denoted λ_{t+1} in Section 2.

Besides the exogenous credit constraint, κ , a “natural” or utility induced self-imposed constraint can be relevant such that the procedure described above should be modified slightly. This is because households want to accumulate enough wealth to buffer against a series of extremely bad income shocks to ensure strictly positive consumption in all periods even in the worst case possible.

Proposition 1. *The lowest possible value of normalized end-of-period wealth consistent with the model, periods prior to retirement, can be calculated as*

$$\underline{a}_t = -\min\{\Omega_t, \kappa\} \forall t \leq T_r - 2$$

where, denoting the lowest possible values of the transitory and permanent income shock as $\underline{\varepsilon}$, and $\underline{\eta}$, respectively, Ω_t can be found recursively as

$$\Omega_t = \begin{cases} R^{-1}G_{T_r, \underline{\varepsilon}_{T_r}, \underline{\eta}_{T_r}} & \text{if } t = T_r - 2, \\ R^{-1}(\min\{\Omega_{t+1}, \kappa\} + \underline{\varepsilon}_{t+1})G_{t+1, \underline{\eta}_{t+1}} & \text{if } t < T_r - 2. \end{cases}$$

Proof. Define $\mathbb{E}_t[\cdot]$ as the *worst-case* expectation given information in period t and note that in the last period of working life, $T_r - 1$, households must satisfy $A_{T_r-1} \geq 0$. In the second-to-last period during working life, households must then leave a positive amount of resources in the

worst case possible,

$$\begin{aligned}
\mathbb{E}_{T_r-2}[M_{T_r-1}] &> 0, \\
\mathbb{E}_{T_r-2}[RA_{T_r-2} + Y_{T_r-1}] &> 0, \\
RA_{T_r-2} + G_{T_r-1}P_{T_r-2}\varepsilon_{T_r-1}\eta_{T_r-1} &> 0, \\
&\Downarrow \\
A_{T_r-2} &> \underbrace{-R^{-1}G_{T_r-1}\varepsilon_{T_r-1}\eta_{T_r-1}}_{\equiv \Omega_{T_r-2}}P_{T_r-2}.
\end{aligned}$$

Combining this with the exogenous credit constraint, κ , end-of-period wealth should satisfy

$$A_{T_r-2} > -\min\{\Omega_{T_r-2}, \kappa\}P_{T_r-2}.$$

In period $T_r - 3$, households must save enough to insure strictly positive consumption next period while satisfying the constraint above, in the worst case possible,

$$\begin{aligned}
\mathbb{E}_{T_r-3}[M_{T_r-2}] &> -\min\{\Omega_{T_r-2}, \kappa\}\mathbb{E}_{T_r-3}[P_{T_r-2}], \\
RA_{T_r-3} + G_{T_r-2}P_{T_r-3}\varepsilon_{T_r-2}\eta_{T_r-2} &> -\min\{\Omega_{T_r-2}, \kappa\}G_{T_r-2}P_{T_r-3}\eta_{T_r-2}, \\
&\Downarrow \\
A_{T_r-3} &> \underbrace{-R^{-1}(\min\{\Omega_{T_r-2}, \kappa\} + \varepsilon_{T_r-2})G_{T_r-2}\eta_{T_r-2}}_{\equiv \Omega_{T_r-3}}P_{T_r-3},
\end{aligned}$$

such that end of period wealth in period $T_r - 3$ should satisfy

$$A_{T_r-3} > -\min\{\Omega_{T_r-3}, \kappa\}P_{T_r-3}.$$

Hence, we can find Ω_t recursively by the formula in Proposition 1 and calculate the lowest value of the grid of normalized end-of-period wealth as $\underline{a}_t = -\min\{\Omega_t, \kappa\}$. \square

D Uncovering Permanent Income Using the Kalman Filter

Here, I give a brief description of the implementation of the Kalman Filter. See, e.g., [Hamilton \(1994, ch. 13\)](#) for a detailed description of the Kalman Filter for time series processes. Formulating the log income process, on State Space form yields

$$\begin{aligned}\mathbf{z}_{it} &= \mathfrak{A} + \mathfrak{B}\mathbf{x}_{it} + \mathbf{v}_{it}, \\ \mathbf{x}_{it} &= \mathfrak{C}_t + \mathfrak{D}\mathbf{x}_{it-1} + \mathbf{u}_{it},\end{aligned}$$

where

$$\begin{aligned}\mathbf{z}_{it} &= \log Y_{it}, \quad \mathfrak{A} = -\frac{1}{2}\sigma_\varepsilon^2, \quad \mathfrak{B} = 1, \quad \mathbf{v}_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2), \\ \mathbf{x}_{it} &= \log P_{it}, \quad \mathfrak{C}_t = -\frac{1}{2}\sigma_\eta^2 + \log G_t, \quad \mathfrak{D} = 1, \quad \mathbf{u}_{it} \sim \mathcal{N}(0, \sigma_\eta^2),\end{aligned}$$

and Y_{it} is observed household income, G_t , σ_ε^2 , and σ_η^2 are known (estimated) parameters and $\log P_{it}$ is the unobserved log permanent income, I wish to uncover. For readability, I suppress i subscripts in what follows.

The Kalman Filter consists of a *prediction step* and an *updating step* where - given initial values, that I discuss below - the prediction step for the process at hand is,

$$\begin{aligned}\mu_{t|t-1} \equiv \mathbb{E}[\mathbf{x}_t | \mathfrak{S}_{t-1}] &= \mathfrak{C}_t + \mathfrak{D}\mu_{t-1|t-1}, \\ &= -\frac{1}{2}\sigma_\eta^2 + \log G_t + \mu_{t-1|t-1}, \\ \Sigma_{t|t-1} \equiv \mathbb{V}[\mathbf{x}_t | \mathfrak{S}_{t-1}] &= \mathfrak{D}\Sigma_{t-1|t-1}\mathfrak{D}' + \sigma_\eta^2, \\ &= \Sigma_{t-1|t-1} + \sigma_\eta^2,\end{aligned}$$

where \mathfrak{S}_s denotes information known at time s . The updating step is given by

$$\begin{aligned}\mu_{t|t} \equiv \mathbb{E}[\mathbf{x}_t | \mathfrak{S}_t] &= \mu_{t|t-1} + K_t(\mathbf{z}_{it} - \mu_{t|t-1} - \mathfrak{A}), \\ &= \mu_{t|t-1} + K_t(\log Y_t - \mu_{t|t-1} + \frac{1}{2}\sigma_\varepsilon^2), \\ \Sigma_{t|t} \equiv \mathbb{V}[\mathbf{x}_t | \mathfrak{S}_t] &= (I - K_t\mathfrak{B})\Sigma_{t|t-1}, \\ &= (I - K_t)\Sigma_{t|t-1},\end{aligned}$$

where $\mu_{t|t} = \log \hat{P}_t$ is the “estimated” log permanent income and K_t is the *Kalman gain*,

$$K_t = \Sigma_{t|t-1}(\Sigma_{t|t-1} + \sigma_\varepsilon^2)^{-1}.$$

For each household, I identify the first year observed in the data (denoted $t = 0$) and use that observation as initial values for $\mu_{t|t}$ and $\Sigma_{t|t}$. Specifically, I assume that log income is at its population mean when first observed in the data, $\log Y_0 = \mathbb{E}[\log P_0 - \frac{1}{2}\sigma_\varepsilon^2 + \mathbf{v}_t | \mathfrak{S}_t]$, such that $\mu_{0|0} = \log Y_0 + \frac{1}{2}\sigma_\varepsilon^2$ and $\Sigma_{0|0} = \sigma_\varepsilon^2$.